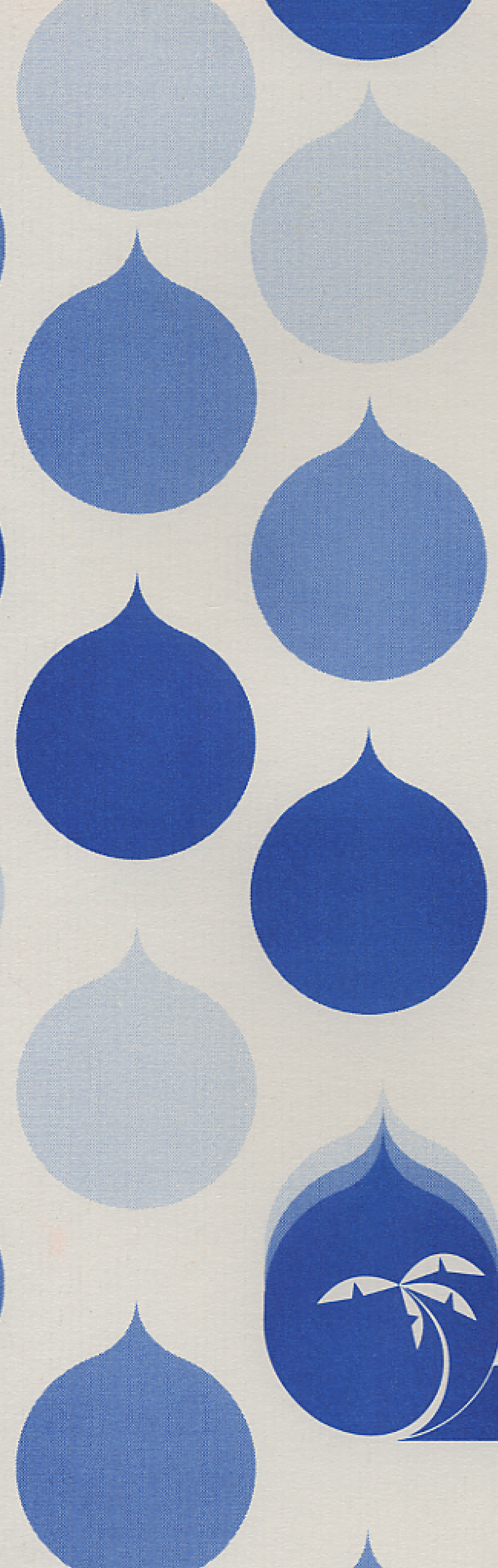

Calibration of a Saltwater Intrusion Model for the Northern Guam Lens Using a Microcomputer

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b	Thickness of Saltwater Layer.
b'_0	Thickness of the Leaky Aquifer.
$\{B\}$	Vector of Nodal Freshwater and Saltwater Thickness.
$[C$	Finite Element Coefficient Matrix for $\{\Phi\}$.
$[D$	Finite Element Coefficient Matrix for $\{\dot{\Phi}\}$.
K_{x_i,x_j}	Permeability Tensor Components
K'	Vertical Permeability in the Leaky Aquifer.
n	Porosity.
$[N$	Matrix of Shape-functions.
N_1, N_2, N_3	Shape-functions for Triangular Element.
q_p	Source/Sink term.
$\{Q\}$	The Right-hand Side(Load Vector) of Finite Element Equation.
R	Residual in the Galerkin Method.
S	Storativity.
Δt	Time increment.
$\{\Phi\}$	Vector of nodal Freshwater and Saltwater Heads.
$\{\dot{\Phi}\}$	Temporal Derivative of $\{\Phi\}$.
γ	Specific Weight of Fluid.
$\Delta\gamma$	$\gamma^s - \gamma^f$
θ	Weighting Function to Obtain Average Values during a Time-step.

Superscipt f represents freshwater and s denotes saltwater.

Abstract

A two-dimensional (areal) finite element model of saltwater intrusion was modified so that it can run on a microcomputer. The model assumes a sharp interface between fresh and salt water and simulates the movement of both fresh and salt water. A preprocessor is provided to remember the nodes of a given network so as to reduce the bandwidth of the matrix. Linear triangular elements are used to discretize the domain. The model was applied to the Northern Guam aquifer. The hydraulic conductivity in three regions of the aquifer was calibrated using the water level history in a few observation wells. Field data on the depth of the interface indicate the sharp interface assumption is valid in the major portion of the aquifer. Comparison of the depth of the measured 50% isochlor with the computed depth of the interface shows that the two are equal in most locations. In some cases, the computed depth is less than the measured depth, resulting in a conservative estimate of the interface depth.

Key Words: Saltwater Intrusion; 2-D Modeling; Microcomputer Application; Sharp Interface depth; Field Verification.

1 Introduction

For careful management and efficient planning of a coastal aquifer, it is essential to predict the sea water intrusion into the aquifer in response to variations in the components of the freshwater balance. The need for an accurate model to simulate the behavior of coastal aquifers is made more acute by the fact that many of the coastal areas are heavily urbanized and therefore require an adequate and sustained yield of water which is fit for drinking and other purposes. Various numerical models have been used in the past to estimate the location of the salt water interface for a given set of hydrologic conditions. These models, depending upon the way they treat the interface, are broadly classified into two types: (a) diffused interface models and (b) sharp interface models. In the first type of model [1,2], the zone of contact between freshwater and seawater is a transition zone, caused by hydrodynamic dispersion, across which the density of the mixed water varies from that of freshwater to that seawater. The second type of model [3,4,5] approximates this gradual transition from freshwater to seawater by sharp interface. In general, there is always some degree of salt water diffusion in coastal aquifers. However, under certain conditions, the width of the transition zone is small relative to the thickness of the aquifer, so that it may still be appropriate to use a sharp interface model. The assumption of an abrupt interface, especially when combined with certain assumptions related to horizontal flow, greatly simplifies the model in most cases of practical interest.

Several numerical methods, including the method of characteristics, finite difference methods, and finite element methods, have been used in sharp interface models. In this study, the governing equations are solved by using the finite element discretization of domain into triangular elements and applying the method of weighted residuals with the Galerkin scheme. This model draws heavily from a previous model developed by Contractor [4], with some improvements in computer storage, computation time and graphical display. A band-width minimization algorithm has been added to arrive at an efficient node-numbering scheme and the stiffness matrix has been manipulated to make it symmetric. The model has been applied to a variety of flow problems with known analytic solutions to confirm its numerical accuracy. It was then used to model the behavior of the aquifer in Guam and results were compared with the observed values of the freshwater elevation and the interface depth.

2 Description of Preprocessor

The saltwater intrusion program requires the flow domain (aquifer in plan) to be discretized into triangular elements. The elements and the nodes of the network are then numbered. In the solution of the freshwater and saltwater heads at the nodes of the network, the program has to solve a matrix of equations. The solution of this set of equations is time-consuming and requires considerable computer storage space. Both the computer storage space and execution time are a function of the bandwidth of the matrix. The bandwidth is a function of the numbering of the nodes and is independent of the numbering of the elements. Reduction in the bandwidth results in a reduction of storage space and execution time. The preprocessor is a program that takes a network with an initial numbering of the nodes and then renumbers the nodes so that the bandwidth is minimized.

The program is based on an algorithm suggested by Collins, [9]. This procedure starts with a node in the initial network and renumbers it as node ♯1 in the new network. Then, the program determines the nodes connected to it and renumbers them as new nodes 2,3,4, etc.. The program takes new node ♯2 finds out the nodes it is connected to and renumbers them in sequence. When all the nodes in the network are thus renumbered, the bandwidth of the system is determined and compared with the initial bandwidth. If the bandwidth has been reduced, the new numbering system is stored in memory, otherwise it is discarded. This procedure is repeated starting with every node in the initial network. The output of the program presents the old node number and the new node number for every node in the network. The optimized network can then be used for production runs with the main program. The preprocessor needs to be run only when changes in the network are made.

This program was applied to the network used for the Guam Lens, [4]. The bandwidth was reduced from 83 to 63 and the execution time was reduced by 10% for each solution of the matrix. Appendix A contains a listing of the preprocessor and the output of the program when applied to the Guam Lens. See fig.1.

3 Governing Differential Equations

The depth-averaged differential equations for freshwater and saltwater have been derived by Sa Da Costa and Wilson [5]. Their derivation involved the following assumptions:

1) Darcy's law is valid

2) A sharp interface exists between the freshwater and saltwater.

3) The horizontal permeability varies negligibly in the vertical direction.

4) The Dupuit approximation holds for unconfined aquifers and essentially horizontal flow occurs in confined aquifers. Only recharge or leakage will occur in the vertical direction.

5) Both freshwater and saltwater are homogeneous and isotropic fluids with constant density and viscosity.

6) Saturated flow occurs in the fresh water and saltwater zones.

Freshwater:

$$\frac{\partial}{\partial x_i}(K^f_{x_i x_j} b^f \frac{\partial \phi^f}{\partial x_j}) + N + q^f_p - \frac{K'}{b'_0}(\phi^f - \phi_0) = (S^f + \frac{n\gamma^f}{\Delta\gamma})\frac{\partial \phi^f}{\partial t} - \frac{n\gamma^s}{\Delta\gamma}\frac{\partial \phi^s}{\partial t}$$

Saltwater:

$$\frac{\partial}{\partial x_i}(K^s_{x_i x_j} b^s \frac{\partial \phi^s}{\partial x_j}) + q^s_p = (S^s + \frac{n\gamma^s}{\Delta\gamma})\frac{\partial \phi^s}{\partial t} - \frac{n\gamma^f}{\Delta\gamma}\frac{\partial \phi^f}{\partial t}$$

Einstein's summation convention is used with i,j = 1,2.

For this report, a few more assumptions were made to simplify the differential equations still further:

1) Soil and water compressibilities are small, so that S^f and S^s can be dropped from the right-hand side of the equations.

2) $q^s_p = 0$ i.e. there is no saltwater source or sink.

3) x_1 and x_2 are the principal directions of the ellipse of permeabilities such that

$$K_{x_i x_j} = 0 \qquad if \quad i \neq j$$
$$K_{x_i x_j} = K_{x_i} \qquad if \quad i = j$$

To make the final element matrix symmetric, the saltwater equation needs to be multiplied by γ^s/γ^f.

Thus, the final differential equations are:

$$\frac{\partial}{\partial x_1}(K^f_{x_1} b^f \frac{\partial \phi^f}{\partial x_1}) + \frac{\partial}{\partial x_2}(K^f_{x_2} b^f \frac{\partial \phi^f}{\partial x_2}) + N + q^f_p - \frac{K'}{b'_0}(\phi' - \phi_0)$$

$$= \frac{n\gamma^f}{\Delta\gamma}\frac{\partial \phi^f}{\partial t} - \frac{n\gamma^s}{\Delta\gamma}\frac{\partial \phi^s}{\partial t} \tag{1}$$

$$\frac{\gamma^s}{\gamma^f}\{\frac{\partial}{\partial x_1}(K^s_{x_1} b^s \frac{\partial \phi^s}{\partial x_1}) + \frac{\partial}{\partial x_2}(K^s_{x_2} b^s \frac{\partial \phi^s}{\partial x_2})\} = n\frac{\gamma^s}{\gamma^f}\frac{\gamma^s}{\Delta\gamma}\frac{\partial \phi^s}{\partial t} - \frac{n\gamma^s}{\Delta\gamma}\frac{\partial \phi^f}{\partial t} \tag{2}$$

The elevation of the interface can be determined from the continuity of pressure across the interface, ie. $p^f = p^s$ at $z = \zeta$. Thus

$$(\phi^f - \zeta)\gamma^f = (\phi^s - \zeta)\gamma^s \qquad or \qquad \zeta = \frac{\gamma^s\phi^s - \gamma^f\phi^f}{\Delta\gamma} \tag{3}$$

Equations (1) and (2) are solved numerically using the finite element method for ϕ^f and ϕ^s at the nodes of the element network. Equation (3) is then used to determine the elevation of the interface at each node.

4 Finite Element Formulation

Linear triangular elements are used to discretize the domain of the aquifer in plan. Let N_1, N_2, N_3 be the usual linear shape functions for a triangular element and let $\{\Phi\}$ be the vector of nodal degrees of freedom for such an element.

Thus,

$$\{\Phi\} = [\phi^f_1 \ \phi^s_1 \ \phi^f_2 \ \phi^s_2 \ \phi^f_3 \ \phi^s_3]^T \tag{4}$$

Let

$$\phi^f = [N^f]\{\Phi\} \qquad and \qquad \phi^s = [N^s]\{\Phi\} \tag{5}$$

Where $[N^f] = \{N_1 \ 0 \ N_2 \ 0 \ N_3 \ 0\}$ and $[N^s] = \{0 \ N_1 \ 0 \ N_2 \ 0 \ N_3\}$

Similarly, we can define

$$\{B\} = [b^f_1 \ b^s_1 \ b^f_2 \ b^s_2 \ b^f_3 \ b^s_3]^T \tag{6}$$

Then,

$$b^f = [N^f]\{B\} \ \ and \ \ b^s = [N^s]\{B\} \tag{7}$$

Substituting these expressions in equations (1) and (2), we define the residuals as

$$R^f = \frac{\partial}{\partial x_1}\{K^f_{x_1}[N^f]\{B\}\frac{\partial}{\partial x_1}[N^f]\{\Phi\}\} + \frac{\partial}{\partial x_2}\{K^f_{x_2}[N^f]\{B\}\frac{\partial}{\partial x_2}[N^f]\{\Phi\}\} + N$$

$$+ q_s^f - \frac{K'}{b'_0}([N^f]\{\Phi\} - z_0) - \frac{n\gamma^f}{\Delta\gamma}[N^f]\{\dot{\Phi}\} + \frac{n\gamma^s}{\Delta\gamma}[N^s]\{\dot{\Phi}\} \qquad (8)$$

$$R^s = \frac{\gamma^s}{\gamma^f}[\frac{\partial}{\partial x_1}\{K^s_{x_1}[N^s]\{B\}\frac{\partial}{\partial x_1}[N^s]\{\Phi\}\} + \frac{\partial}{\partial x_2}\{K^s_{x_2}[N^s]\{B\}\frac{\partial}{\partial x_2}[N^s]\{\Phi\}\}$$

$$- n\frac{\gamma^s}{\gamma^f}\frac{\gamma^s}{\Delta\gamma}[N^s]\{\dot{\Phi}\} + \frac{n\gamma^s}{\Delta\gamma}[N^f]\{\dot{\Phi}\} \qquad (9)$$

The Galerkin weighted-residual method utilizes the following procedure to obtain the element equations.

$$\int_A N_i R^f dA = 0 \;\; and \;\; \int_A N_i R^s dA = 0 \;\; for \;\; i = 1,2,3. \qquad (10)$$

These six equations can be written in matrix form as follows:

$$[C]\{\Phi\} + [D]\{\dot{\Phi}\} = \{Q\} + Boundary \;\; terms \qquad (11)$$

The matrices [C] [D] and $\{Q\}$ are derived in Appendix D. The boundary terms are also derived for those elements that have one or more sides along the boundary.

A time-integration scheme is adopted and a weighing factor θ is defined. Thus,

$$\{\dot{\Phi}\} = \frac{d}{dt}\{\Phi\} = \frac{\{\Phi\}_{t+\Delta t} - \{\Phi\}_t}{\Delta t} \qquad (12)$$

and

$$\{\Phi\} = (1-\theta)\{\Phi\}_t + \theta\{\Phi\}_{t+\Delta t} \qquad (13)$$

Substituting equations (12) and (13) into (11),
we get

$$[\theta[C] + \frac{1}{\Delta t}[D]]\{\Phi\}_{t+\Delta t} = [-(1-\theta)[C] + \frac{1}{\Delta t}[D]]\{\Phi\}_t + \{Q\}_t$$

$$+ Boundary \;\; terms \;\; averaged \;\; over \;\; \Delta t \qquad (14)$$

It is shown in Appendix D that the matrices [C] and [D] are symmetric. Hence, the matrix $[\theta[C] + \frac{1}{\Delta t}[D]]$ will also be symmetric.

This symmetry results in a reduction of computer storage and execution time, since only the terms on and above the main diagonal need to be saved.

Equation (14) is applied successively to all the elements of the network and a global matrix assembled. The global matrix is solved by a Gauss Elimination subroutine. A few iterations may be required to take care of the non-linearity introduced by the time variation of b^f and b^s.

5 Description of Program

The computer program has the capability to simulate steady and unsteady flows, and analyze both confined and unconfined aquifers. If the aquifer is confined, the program considers leaky or non-leaky conditions. Pumps can be located at any number of nodes of the network. Recharge is assumed constant in an element but can be varied from element to element. Head or flow rates can be applied at the boundaries as a function of time. At a coastal boundary, a mixed or third-type boundary condition can be specified. For steady flow conditions, the saltwater head can be specified to be zero along the boundary or at every node in the network. This procedure assures that the Ghyben-Herzberg condition is satisfied. The program can also be run in the unsteady mode with the Ghyben-Herzberg condition. Constant head conditions are taken into account by eliminating that variable from the matrix. The righthand side of each equation is reduced by the product of the constant head times its coefficient in the matrix. The row and column of that variable are eliminated and the size of the matrix reduced.

If the saltwater head at every node in the network is specified to be much less than the anticipated freshwater head, the results of the computer program will show that the thickness of the saltwater layer is equal to an arbitrarily small value (BTOE). Under these conditions, the program can simulate flow in a freshwater aquifer. Use of the program to simulate freshwater aquifers will be limited only by the number of nodes in the network, since the element matrix still contains saltwater heads. Apart from this limitation, the solution procedure should still be efficient.

Since the heads are assumed to vary linearly across the triangular element, the velocity will be constant in each element. By specifying NVEL = 1, the program will print out the velocities in the x and y directions in each element in both the freshwater and saltwater layers. These velocities can then be used to calculate the flow rates across any line or boundary. By specifying NTOE = 1, the program will determine where in the network a freshwater or saltwater toe occurs. A saltwater toe occurs at the intersection of the sharp interface with the lower and/or upper boundaries of the aquifer. A freshwater toe occurs at the intersection of the phreatic surface with the lower and/or upper boundaries of the aquifer.

The program assumes that there are two independent variables at each node: the freshwater head and saltwater head. After solving for the heads, equation (3) is used to determine the depth (ζ) of the interface. The location of the interface determines the thickness of the freshwater and the saltwater layers. If, however, the interface is calculated to be below the lower impervious boundary, then the entire aquifer thickness should contain only freshwater and the thickness of the saltwater layer should theoretically be zero. However, the program makes the saltwater layer equal to BTOE. Similarly, when the phreatic surface intersects the lower impervious boundary, the freshwater thickness beyond the toe is made equal to BTOE instead of zero.

Equation (13) introduces a weighting factor, θ, which varies between zero and one. This factor is useful in regulating the stability and accuracy of the solution. When $\theta = 0$, the problem formulation is referred to as explicit In this formulation, the spatial derivatives are evaluated at the known time- step,Δt. The time-step,Δt, necessary for stable results is very small. This results in very long execution times for the program. When $\theta = 0.5$, the approach provides high-accuracy with large values of Δt, even though the results may show some numerical instability. When $\theta = 1.0$, the formulation is known as fully implicit. This formulation provides the maximum stability at a sacrifice of some accuracy. Values of θ between 0.5 and 1.0 (e.g., 0.6, 2/3, 3/4) have been used to provide the proper balance between accuracy and stability. All of the examples presented in this report were run with $\theta = 1$. When steady state results are desired, the program should be run with $\theta = 1.0$ and Δt equal to a very large number (e.g. 1.0E20). Since Δt is large, the time derivative terms in the element equations (1) and (2) become very small. The program should be learn for several time steps because of the non-linear nature of the problem until the error is less than a specified tolerance.

The program can be run in any set of consistent units. Thus, if feet and seconds are the length and time units, the permeability and recharge must be input in ft/sec and the pump rate in cu.ft/sec. If meters and days are the length and time units, then the permeability and recharge must be input in m/day and the pump rate in cu.m/day.

A new feature has been added to simplify the input data requirements. The element network is divided into regions in which the element properties such as conductivity and porosity are constant. Thus, the data for each region needs to be read only once. The same is true for the elevations of the bottom and ceiling of the aquifer at the node of the network.

A listing of the main program (FORTRAN 77) is presented in Appendix B. The user manual describing the input variables and their sequence is given in Appendix C.

6 Comparison of Numerical Results with Analytical Solutions

In order to develop confidence in the program, a few simple simulations were made, for which analytical solutions were available. Van der Veer [10] has given a closed form solution to a one-dimensional flow of freshwater in an unconfined aquifer towards the sea coast. Fig. (2) presents the element network used to simulate the 1-dimensional system. Fig. (3) shows a comparison of the numerical and analytical results. The numerical solution is quite close to the analytical solution. The numerical program was also verified by comparing its output with the analytical solutions of (a) the gravity segregation problem described in reference [5] and (b) the drawdown in an aquifer due to constant-discharge pumping from a well (Theis solution).

7 Application to Northern Guam Lens

Guam is the largest and southernmost of the Mariana Islands. It is approximately 30 miles long and 4 to 10 miles wide and has an area of 212 square miles. It is divided into two nearly equal parts of different geology [6]. The northern half is a broad limestone plateau and the southern half is a dissected volcanic upland. The main aquifer of Northern Guam is bounded by the sea coast on the east, north and west, and by a volcanic outcrop on the south, and comprises ofthe Mariana limestone and the older Barrigada limestone. The aquifer also has a complex basement of impervious volcanic rock. A contour map of the basement surface was prepared based on a study by Biehler and Walen [7] and the basement elevations at the nodes were interpolated from it. The recharge into the aquifer was obtained by using a technique suggested by Mink [8] based upon the contrasting hydrologic features of Northern and Southern Guam.

7.1 Data Base

The available hydrologic data consisted of rainfall records for the years 1978, 79, 80, 85, 86 and 87 and the runoff records for 1978- 1985. The missing data of rainfall for the period 1981-84 was estimated based upon an average rainfall-runoff relation for each month and the available runoff records. The runoff for the period 1986-87 was estimated based upon the rainfall values of those years. The pumping rates were observed to be more or less constant from year to year. Therefore the pumping rate for 1981-85 was taken as the average of other years for each month. Ocean level data was available for 78-80 and 85-87. During the years 81-84, the ocean level was assumed to be at mean sea level.

The aquifer was discretized into 295 elements using 189 nodes based upon the requirements of proper accuracy and the limitation of computer storage space. Pump discharges were lumped at 72 nodal points. There were 39 element boundaries along the coastline and the saltwater head was specified at 40 nodes. The porosity of the aquifer was estimated to be 0.25 but the permeability was considered to be varying widely over the area.

7.2 Calibration of Model

The model was calibrated against the observed hydrologic data of 1978,79 and 80. The aquifer was divided into 3 regions of different permeabilities for modeling purposes. See fig.1. Region I contained elements 1 through 24, region II consisted of elements 25 to 83 and Region III comprised the rest of the elements. Several calibration runs were made in which the permeabilities of the three regions were changed until a good comparison was obtained between measured and computed values of water levels at nine observation wells.

Region	Element Nos.	Permeability(ft/day)	Porosity
I	1-24	80	0.25
II	25-83	5000	0.25
III	84-295	20,000	0.25

The average absolute error for these values of permeabilities for all the observation wells (excluding well A-20 which showed a large error which was attributed to some local influence) was about 0.45 feet or about 13%. The error could have been reduced further, if the permeabilities of elements containing observation wells were changed individually but this level of error was considered satisfactory for most planning and management purposes. Therefore no fine-tuning of the model was attempted. Further reduction in the error would be best reduced by utilizing parameter identification and stochastic analyses.

The program used a time step Δt of 1 month. Each iteration (one solution of the global matrix) took approximately 40 secs on the microcomputer. The same program required only one second/iteration on the VAX 11/780.

8 Depth of Sharp Interface

8.1 Measured Data

Exploratory wells were drilled at various locations of the aquifer. The specific conductance of water samples from different depths were measured at six exploratory wells at different times of the year. These measurements were used to plot the chloride- concentration profile with depth at the six wells. These profiles are presented in figs. (4)-(9). The depth of the 50% isochlor is taken to be the depth of sharp interface. From most of the plotted profiles it can be seen that depth over which the chloride concentration varies, is small in absolute dimensions and in relation to the depth below MSL. Thus, the assumption of a sharp interface is reasonable. A notable exception occurs in well Ex-1 which is near region I. See fig. 4. Region I is an argillaceous limestone, in which the conductivity is very low and in which the dispersive effects are large. Fortunately, the areal extent of region I is not large.

8.2 Computed Depths

It was felt desirable to compare the measured interface depths with those calculated by the program. To do this, a simulation run was made for the time period 1978-87 using the hydrologic data base described on page and the calibrated permeabilities (Appendix F). Figs. 10 to 19 show the calculated and measured values of the well water level and the interface depth for the period of simulation. It can be seen that the computed values of the interface depth compare well with the measured values at some of the wells. At the remaining wells, the computed depth is smaller than the measured depth. Thus, the program gives a conservative estimate of the interface depth, the actual depth being greater than the computed one. One of the reasons for the discrepancy is that the program is two-dimensional while the flow is three-dimensional in nature. A three-dimensional program would take into account the vertical components of the velocity, resulting in a higher freshwater head at the interface and hence in a lower elevation of the interface. From the measured values of the interface depth, it can be seen there is no long-term trend, even though there are seasonal changes in the depth. A long-term trend will be experienced if there is a significant increase in the pumping rate from the aquifer.

The computed freshwater head ϕ^f is seen to have many sharp peaks and the measured head is seen to be very smooth and damped. The explanation for this difference lies in the fact that the recharge to the aquifer has to travel through 200-300 feet of unsaturated media before it reaches the groundwater level. The numerical model assumes that the recharge enters the groundwater lens instantly and without change in magnitude. In reality, the recharge will be delayed by the time of travel and the magnitude will be damped by dispersion effects.

The influence of using or neglecting the Ghyben-Herzberg approximation ($\zeta = 40\phi^f$) can be seen in figs. 20-23. Computer runs were made with the saltwater head specified as zero at all nodes of the network. The results of these runs are referred to as static state and are shown in the figures by a dashed line. It can be seen that the influence on the interface depth is only minor. However, the influence on the well water level is quite significant. Using the Ghyben-Herzberg approximation result in the well water levels being damped out, with very little variation in the water levels. The measured data shows more fluctuations in the water level. The influence of the ocean level variations cannot be taken into account when using the Ghyben-Herzberg approximation.

9 Summary and Conclusions

1) A preprocessor has been provided to reduce the bandwidth of a given matrix by re-numbering the nodes of a given triangular, finite element network. Its application to the Guam network decreased the bandwidth from 83 to 63, resulting in a lower storage space requirement for the program and a shorter execution time.
2) A computer program was written in FORTRAN 77 for a microcomputer to simulate two-dimensional (areal) saltwater intrusion into an aquifer. A symmetric matrix was developed to reduce storage space requirements. Improvements have been made to enter data more easily.
3) The numerical program was checked for accuracy against several analytical solutions and a satisfactory agreement between the two solutions was obtained.
4) The program was applied to the Northern Guam Lens. The permeabilities in three regions were calibrated using the measured water level data from nine wells in the aquifer.
5) The measured interface depth data indicated that the assumption of a sharp interface is reasonable in regions II and III. In region I, the permeability is much smaller than in the other regions and the dispersive effects are also greater.
6) A comparison was made between the computed and measured depths of the interface at four exploratory wells. Good agrement was obtained at some of these wells. The computed depths at the other wells were smaller than the measured depth, thus providing a conservative estimate of the interface depth.
7) Using the Ghyben-Herzberg approximation resulted in well water levels that were damped out. Ocean level variations cannot be simulated when using this approximation. Thus, aquifers in which water levels are affected by ocean level fluctuations have to be simulated without the Ghyben-Herzberg approximation. The location of the interface was unaffected by the approximation.

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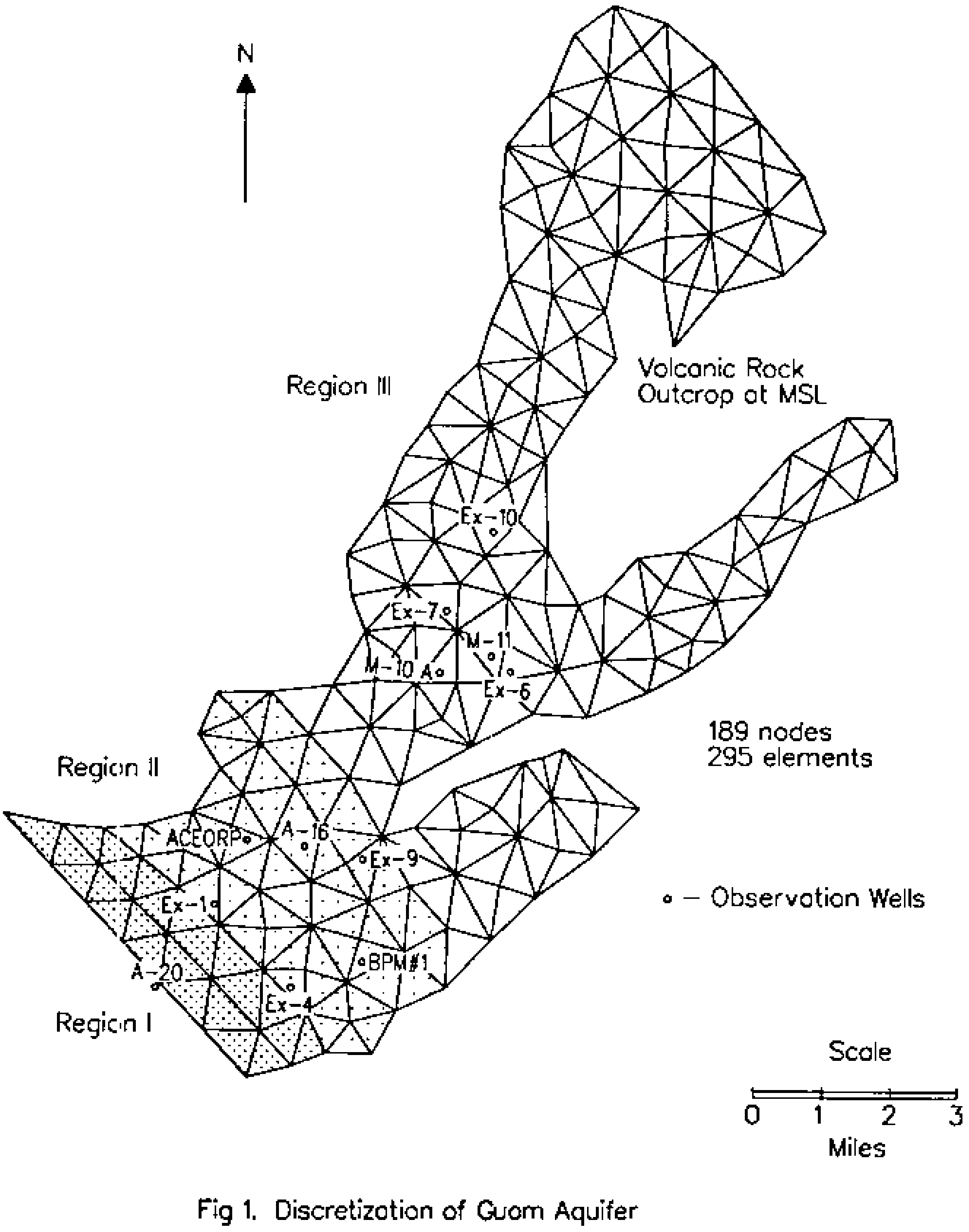

Fig 1. Discretization of Guam Aquifer

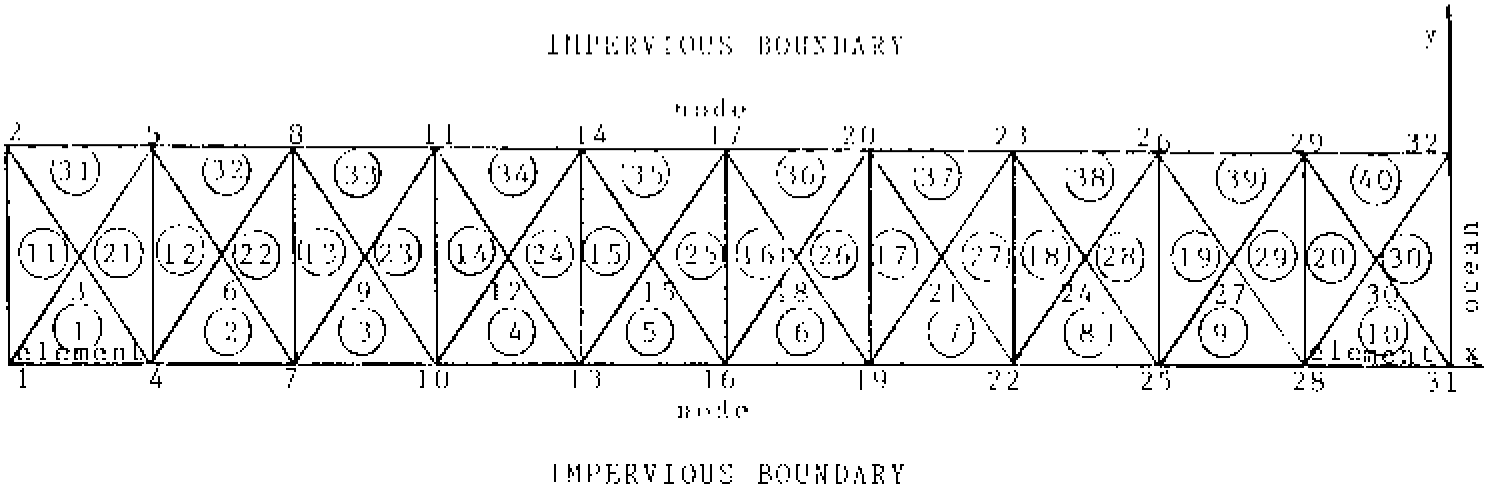

Figure 2. Finite Element Network For 1-D Problem.

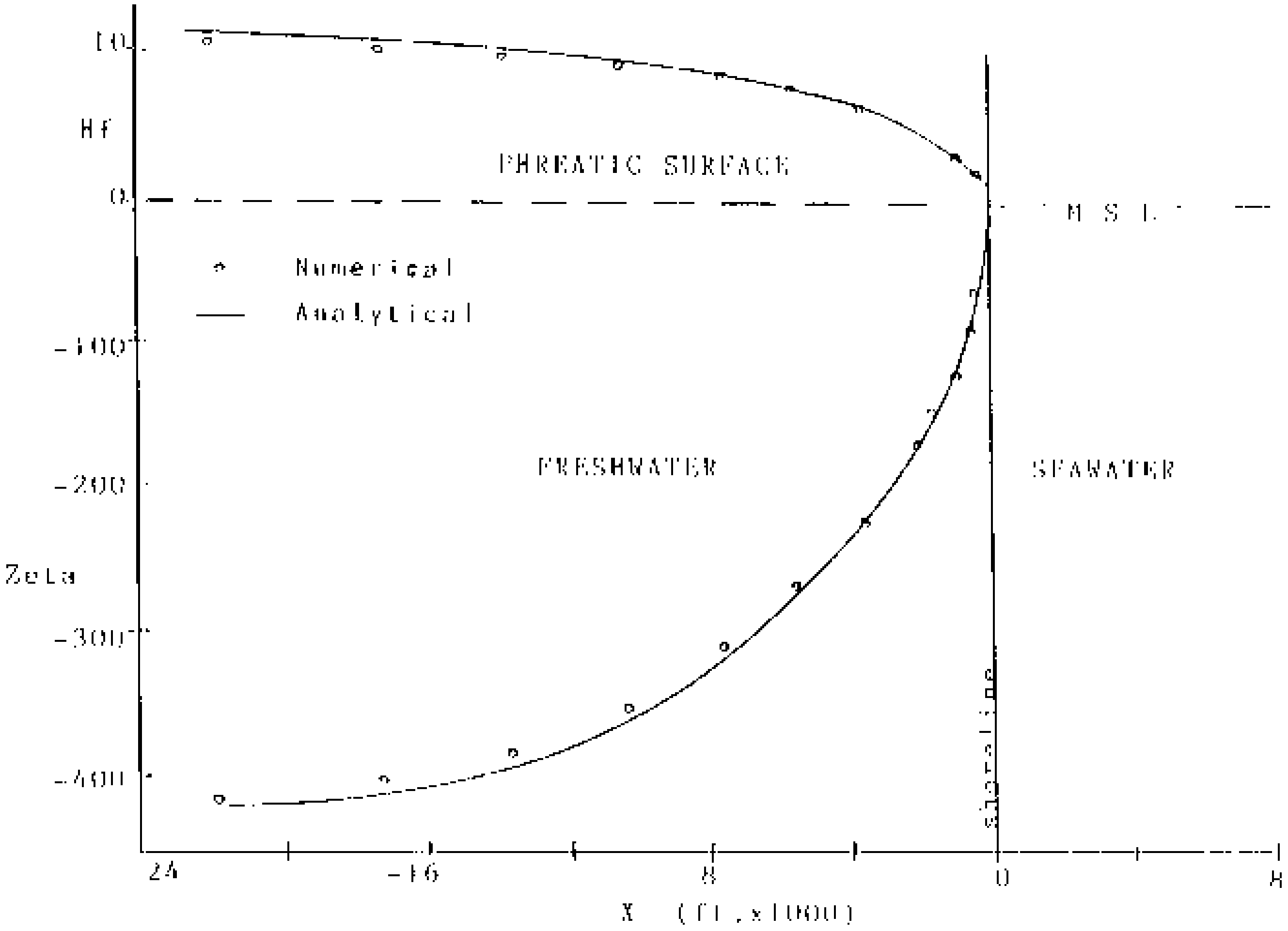

Figure 3. Comparison of numerical and analytical solutions of 1-D problem.

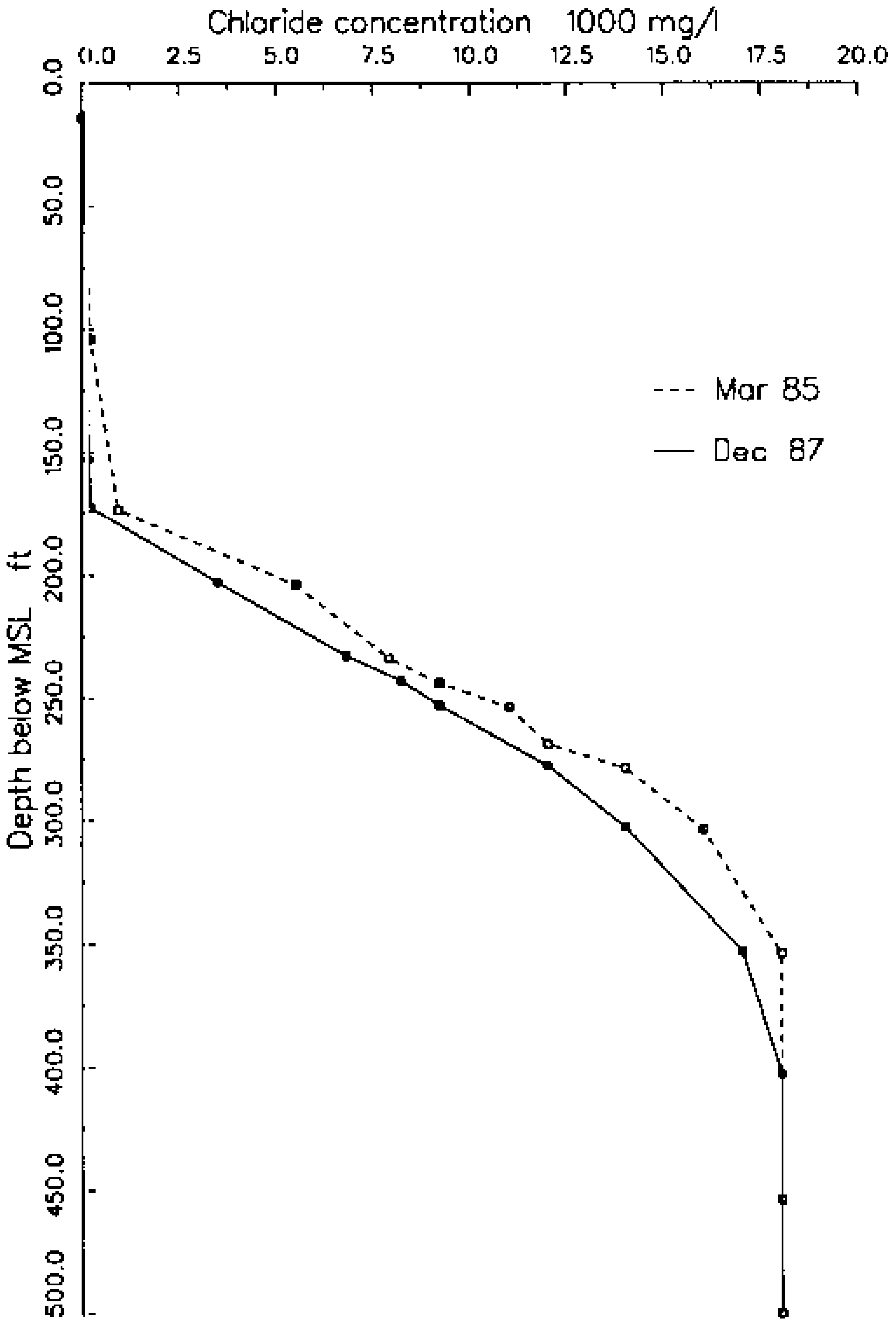

Figure 4. Measured Chloride Concentration Profile At Well Ex-1

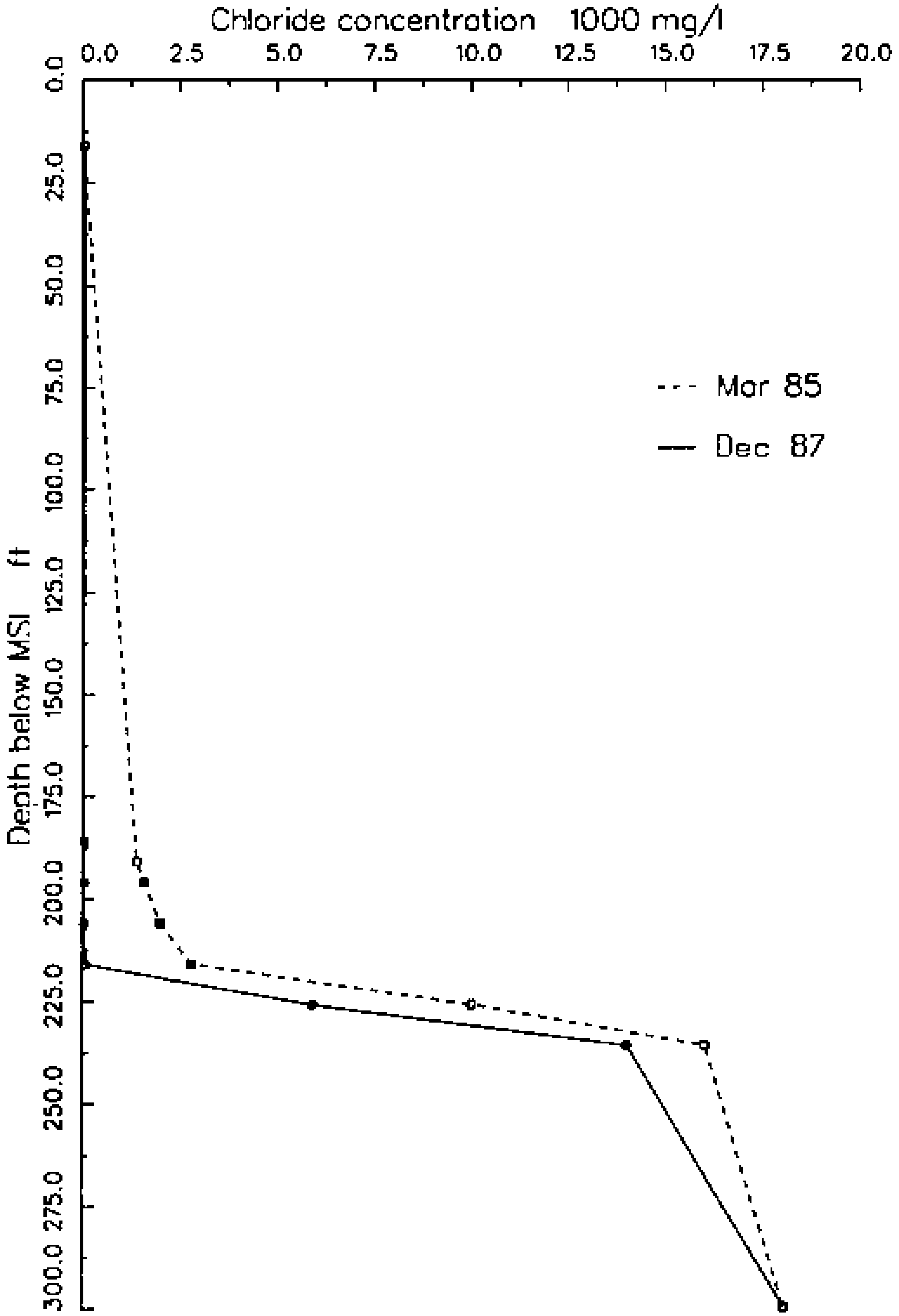

Figure 5. Measured Chloride Concentration Profile At Well Ex-4

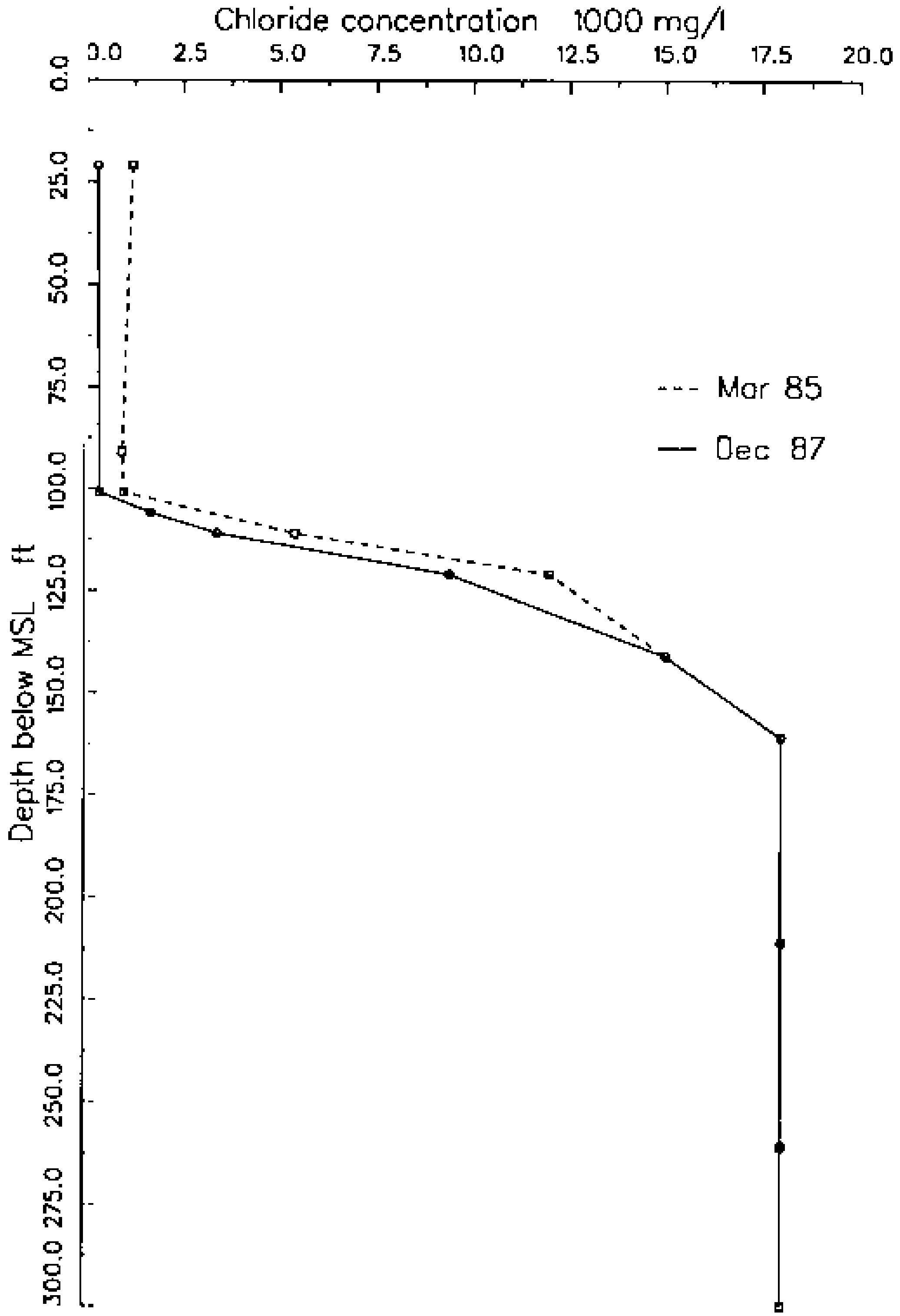

Figure 6. Measured Chloride Concentration Profile At Well Ex-9

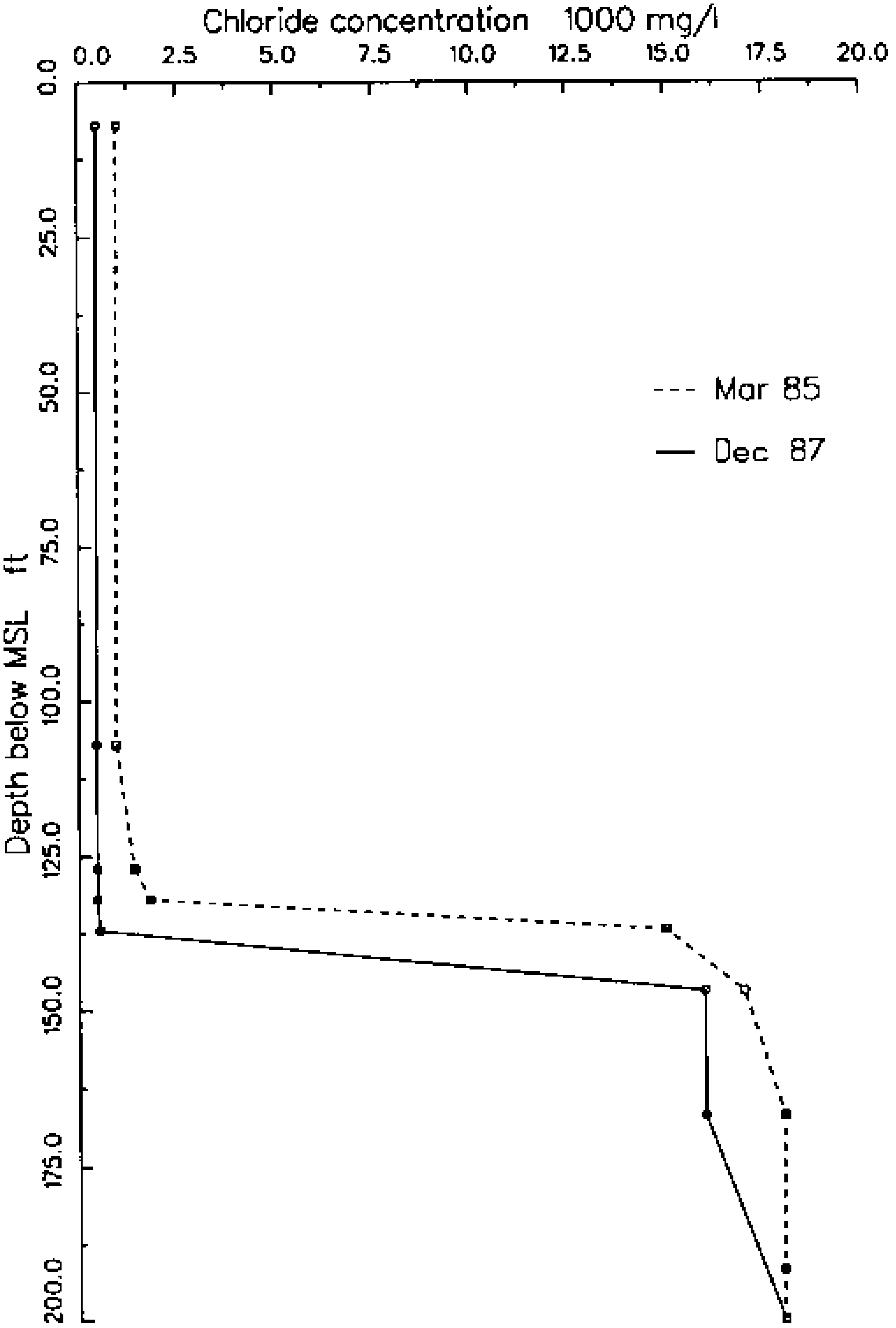

Figure 7. Measured Chloride Concentration Profile At Well Ex-7

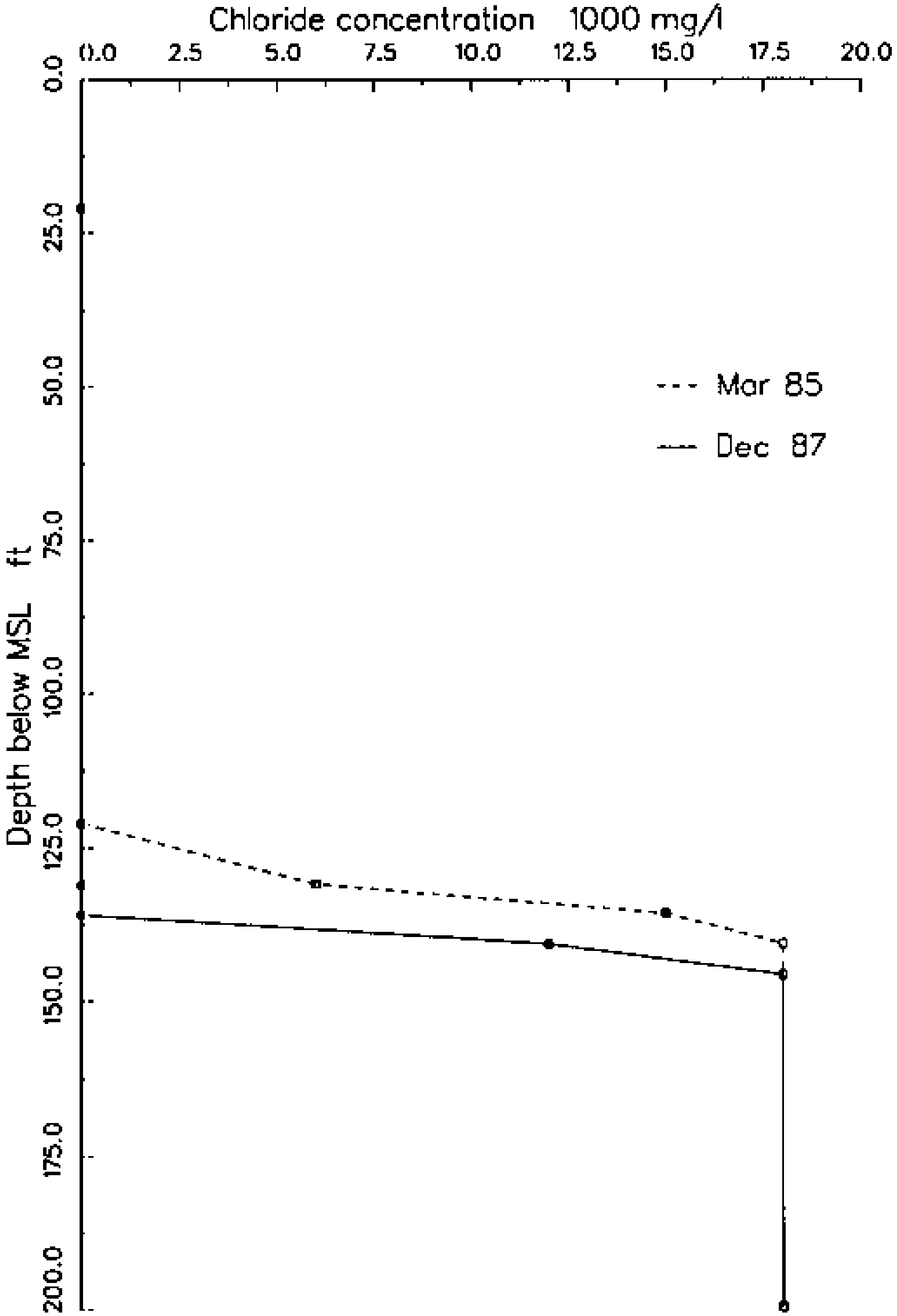

Figure 8. Measured Chloride Concentration Profile At Well Ex-6

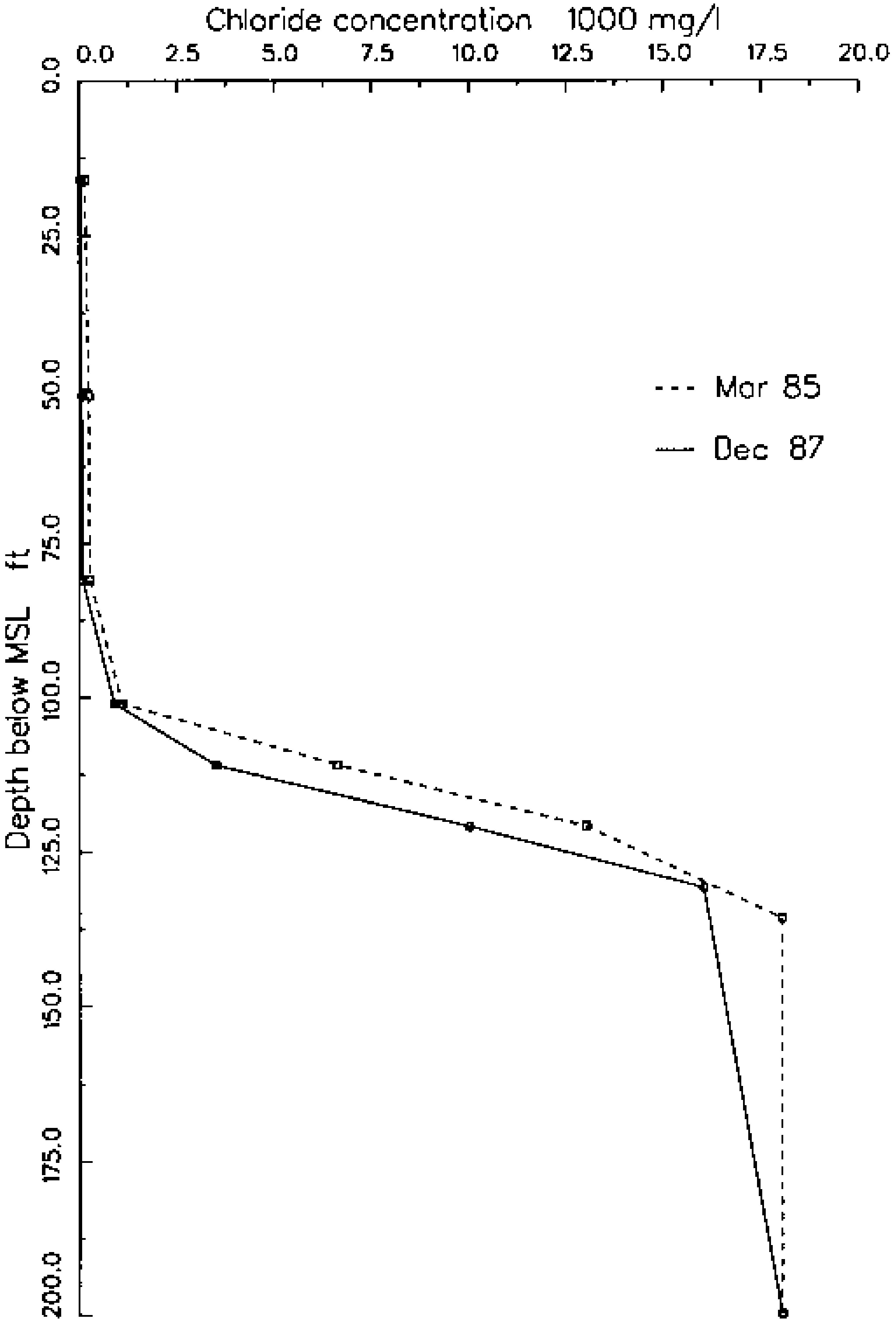

Figure 9. Measured Chloride Concentration Profile At Well Ex-10

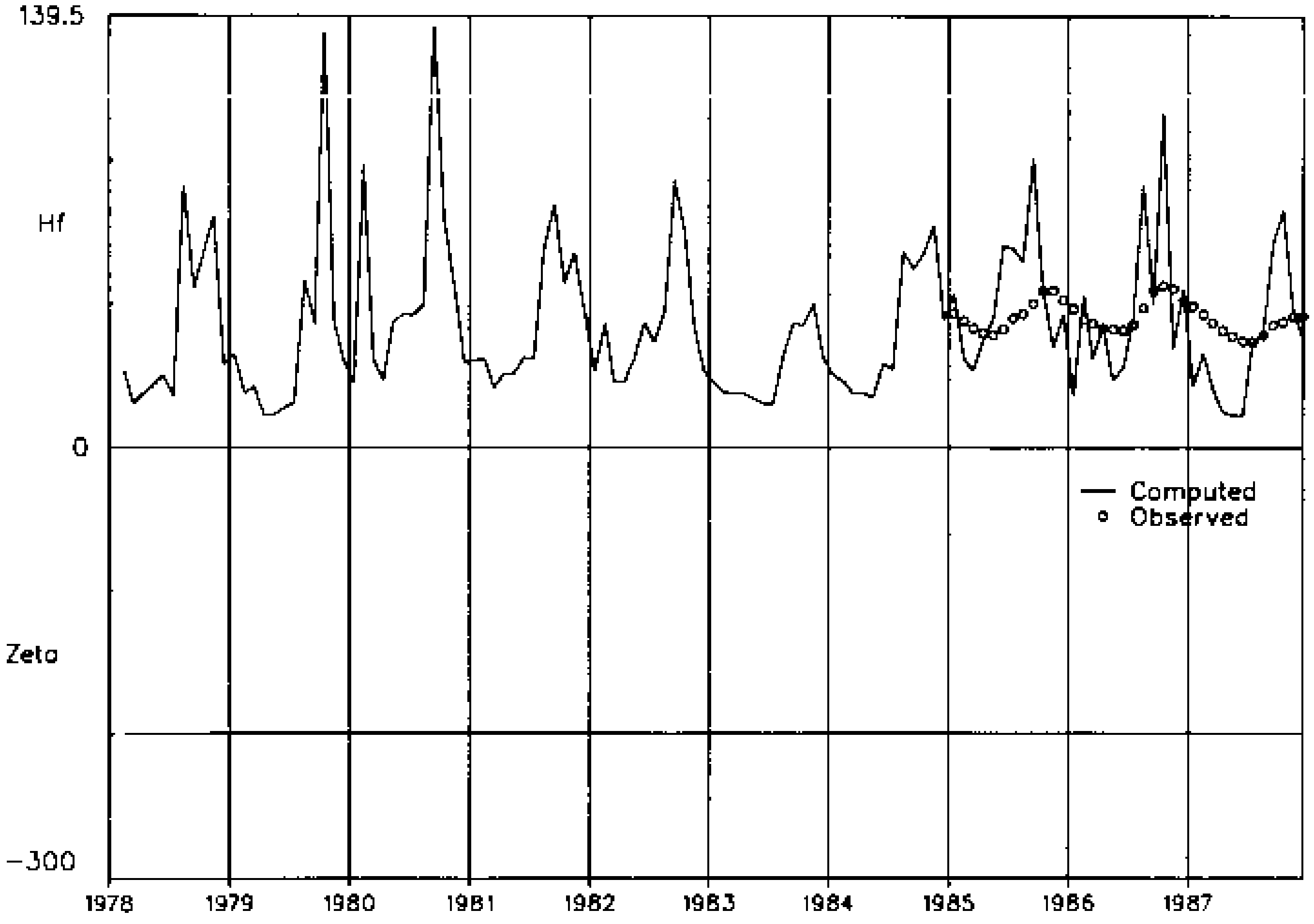

Figure 10. Well water level and interface depth at Well A-20

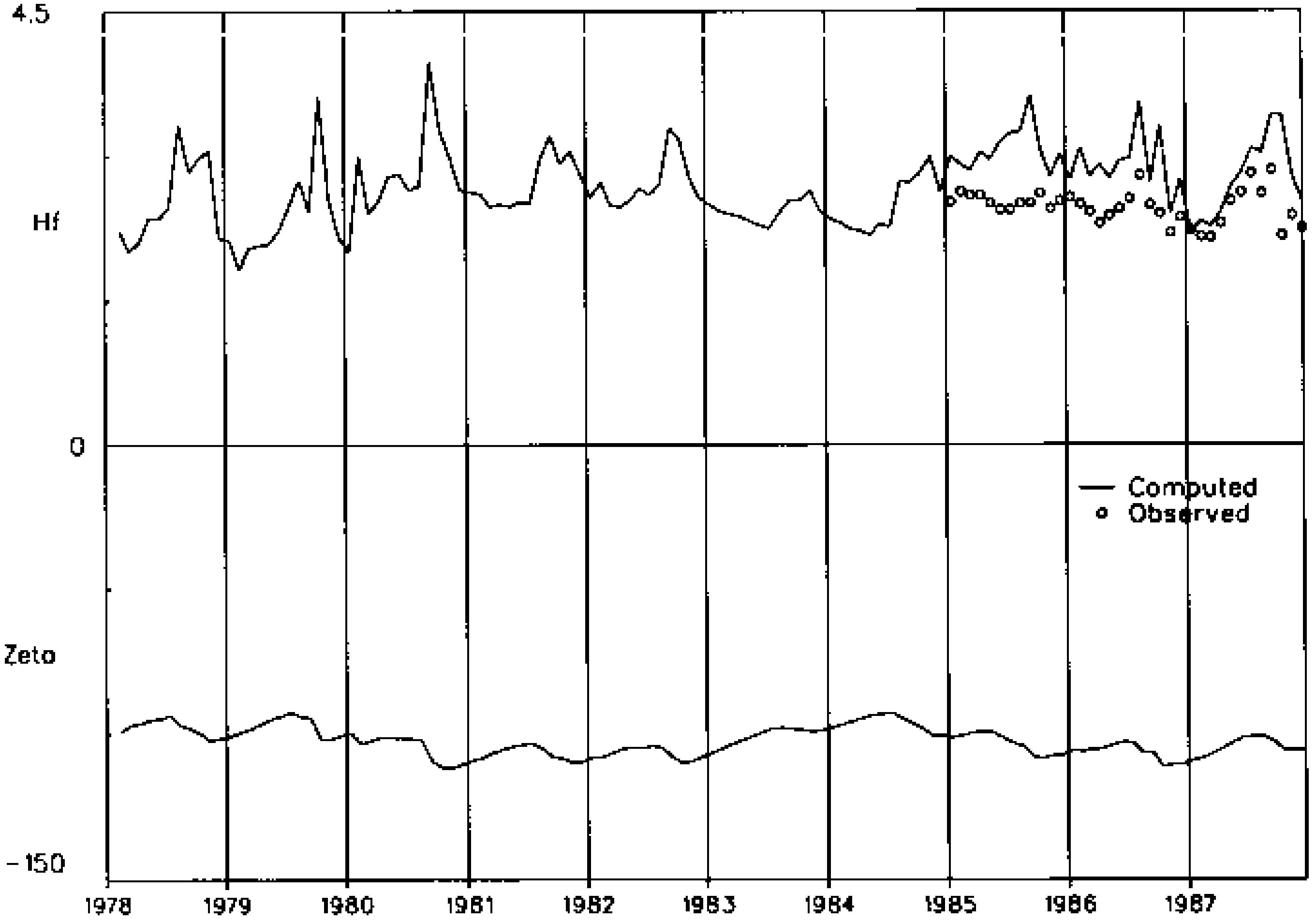

Figure 11. Well water level and Interface depth at Well ACEORP

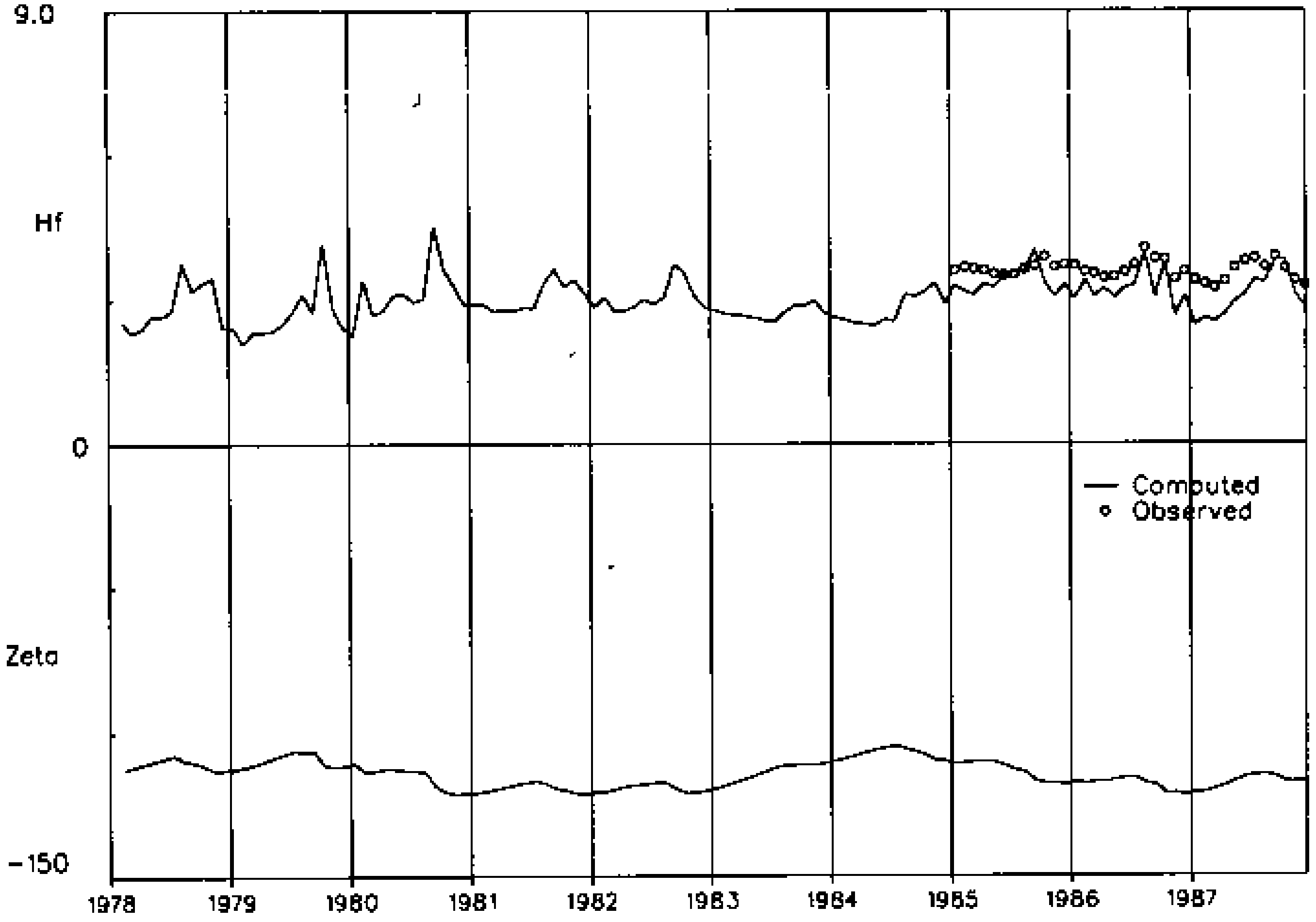

Figure 12. Well water level and Interface depth at Well A-16

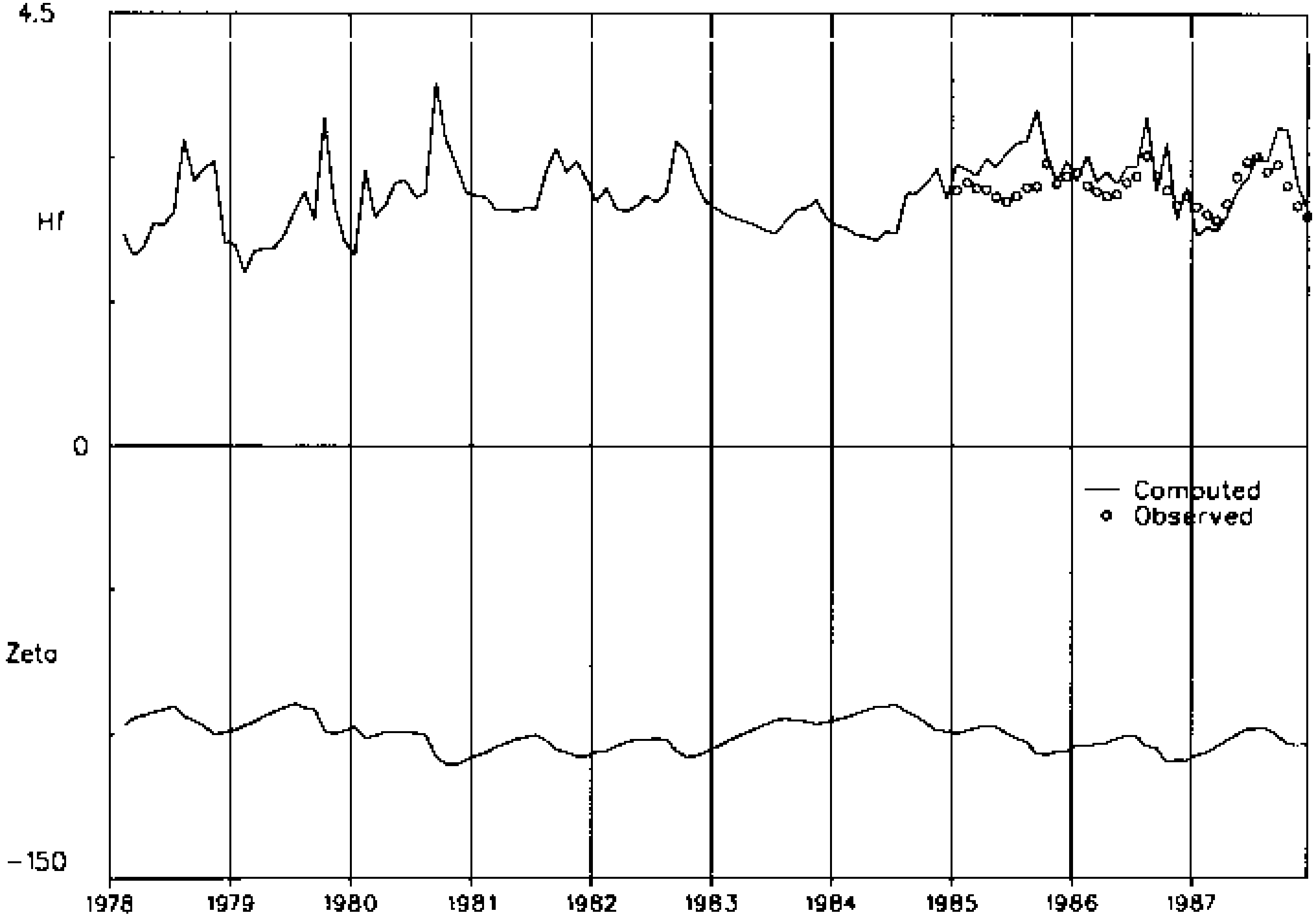

Figure 13. Well water level and Interface depth at Well BPM#1

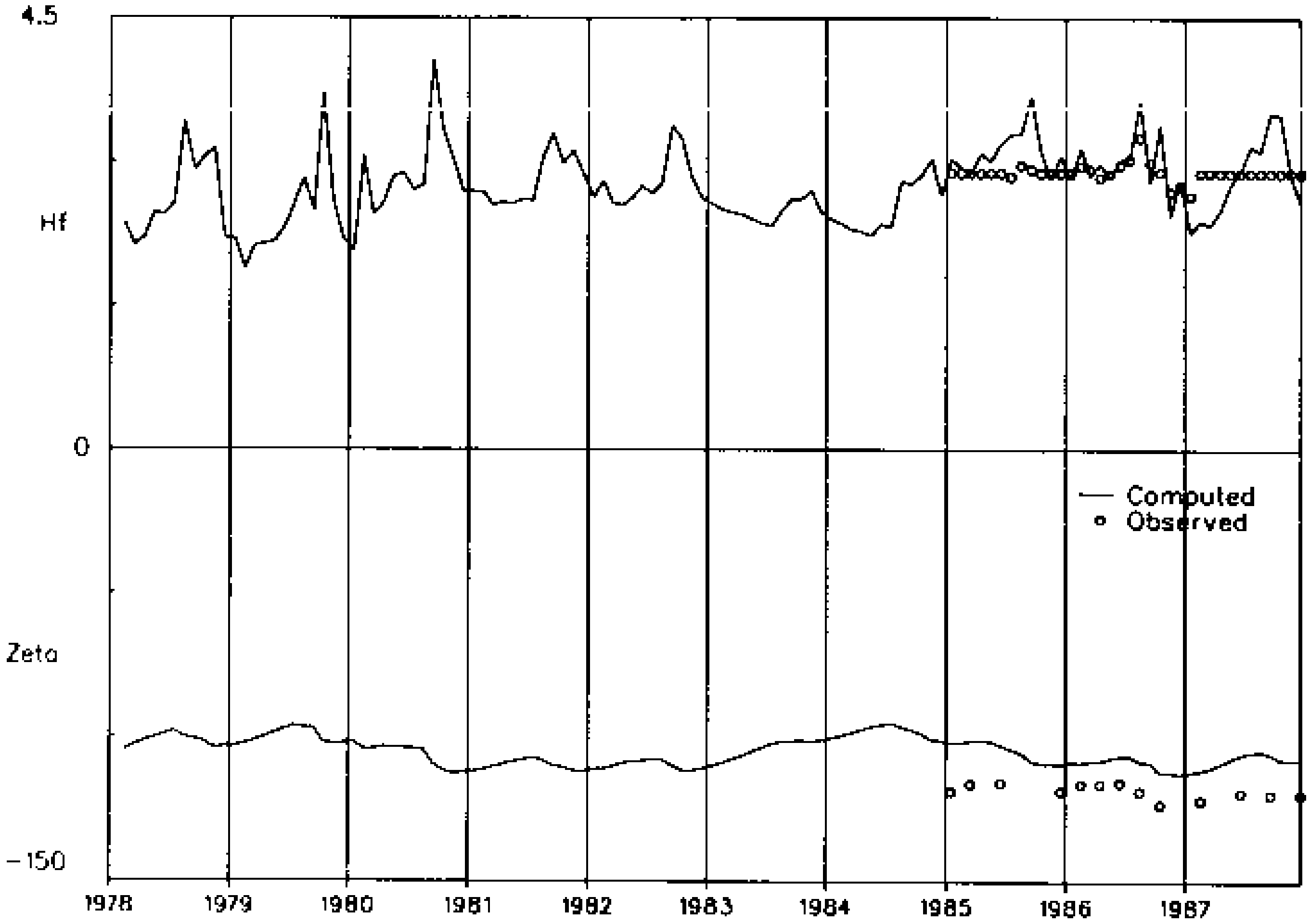

Figure 14. Well water level and Interface depth at Well Ex-9

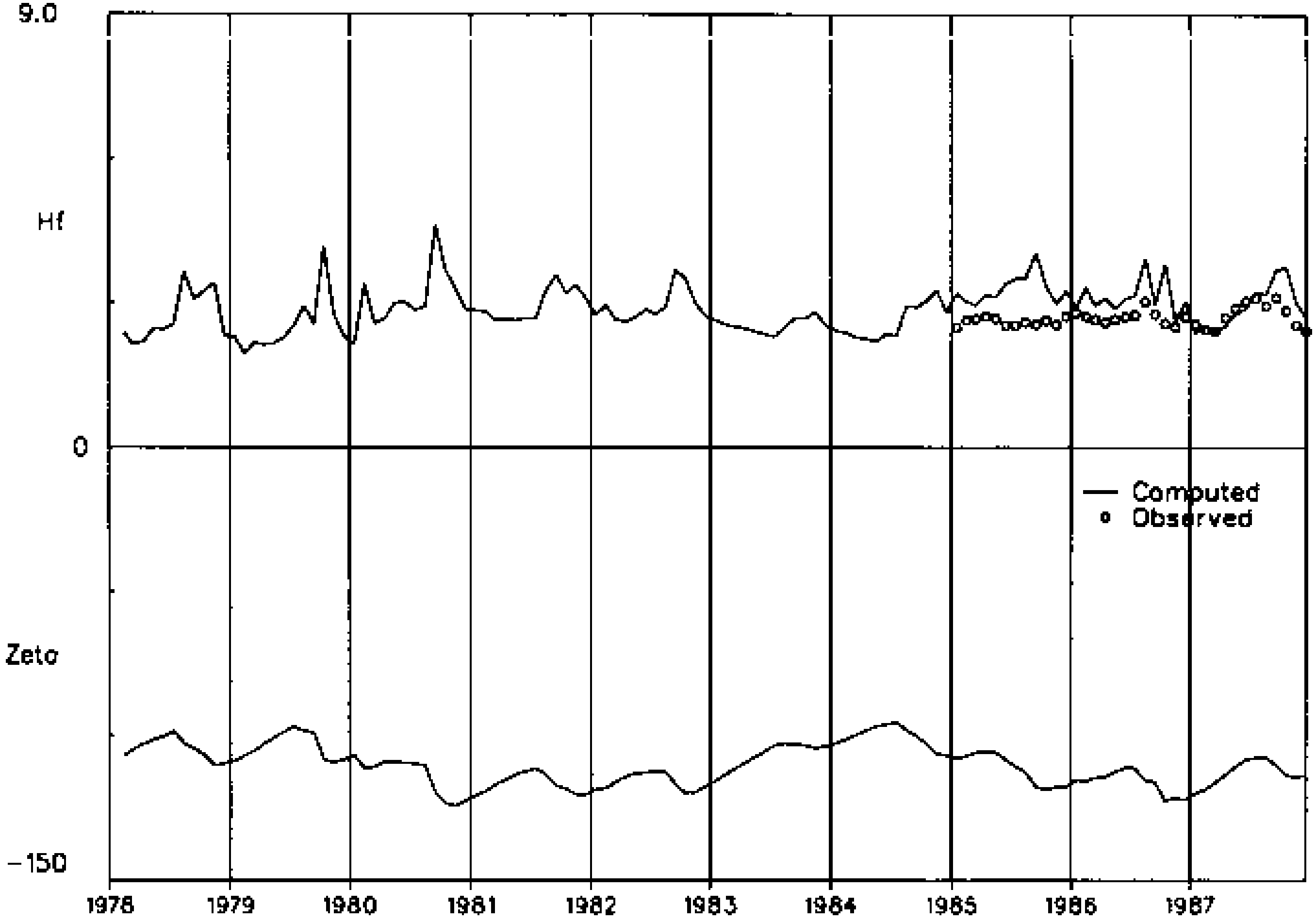

Figure 15. Well water level and Interface depth at Well M-10 A

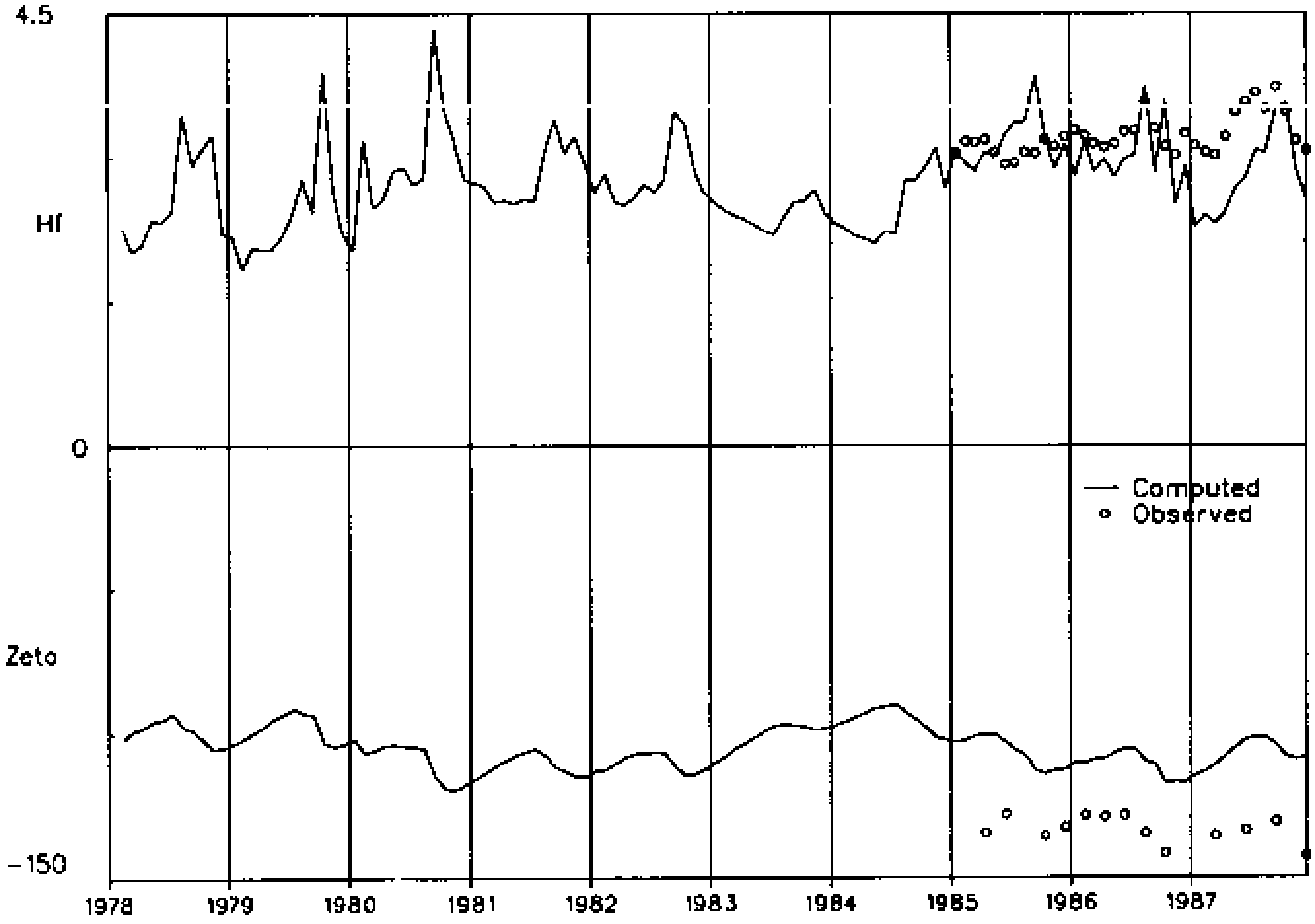

Figure 16. Well water level and interface depth at Well Ex−7

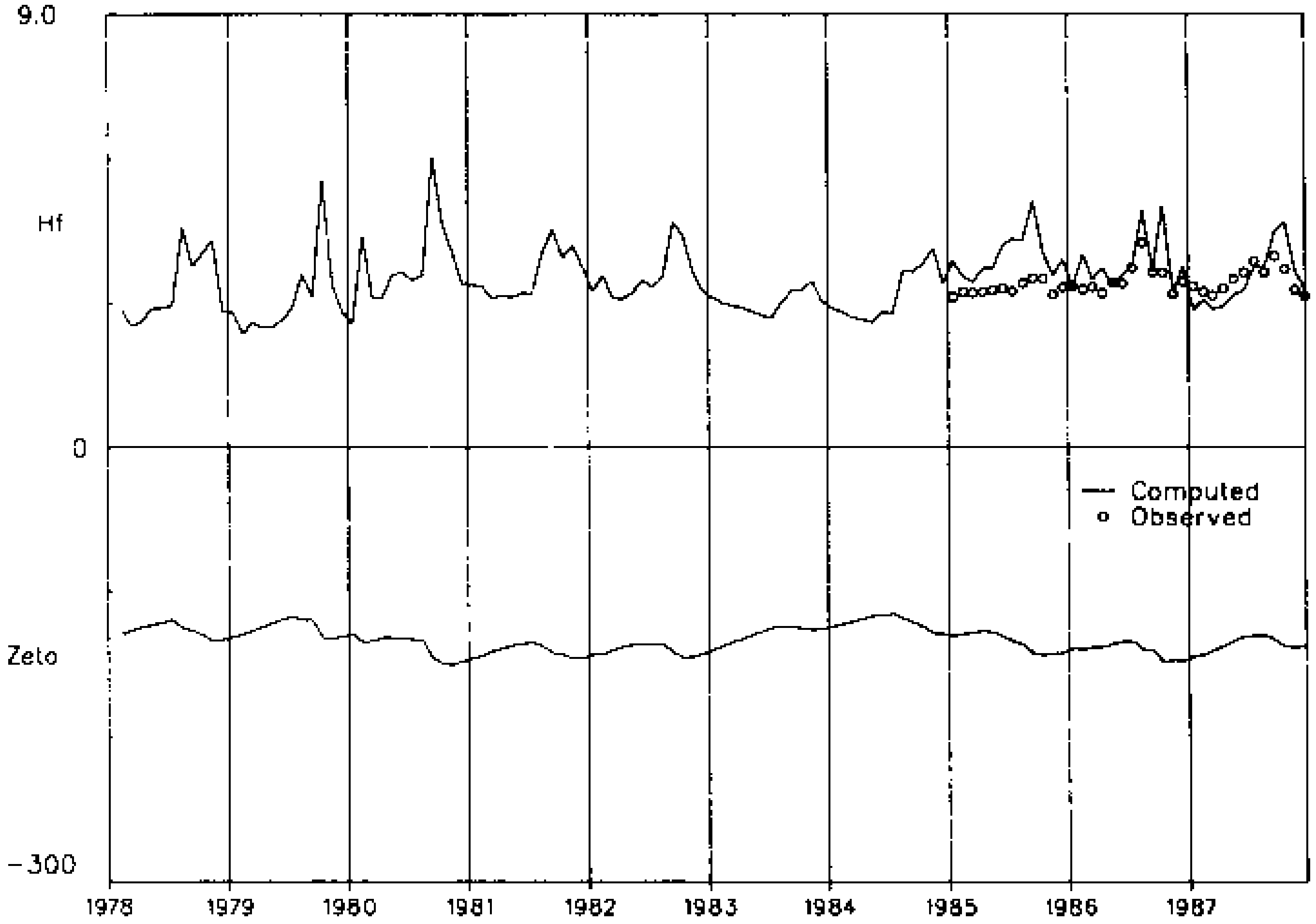

Figure 17. Well water level and Interface depth at Well M-11

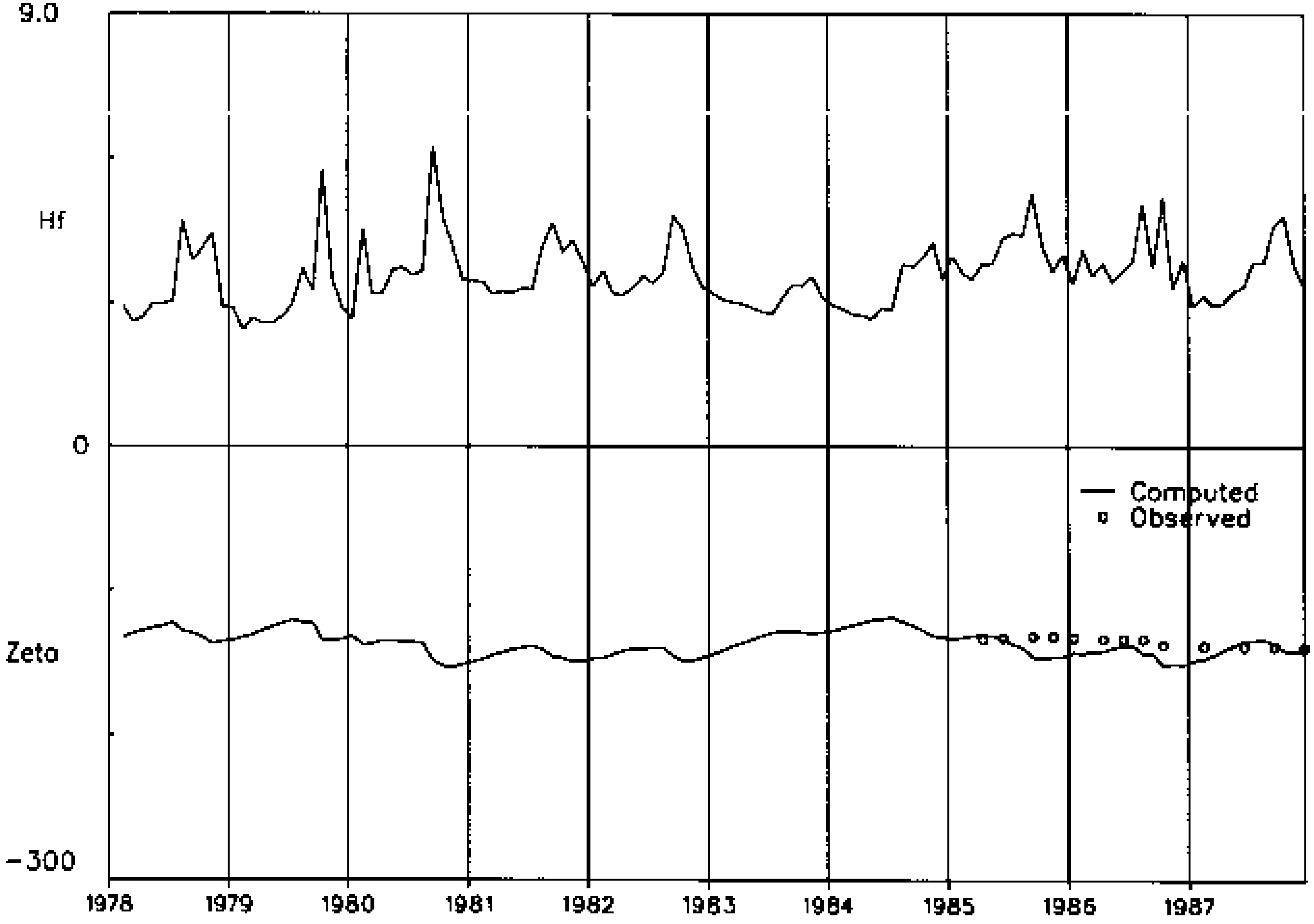

Figure 18. Well water level and Interface depth at Well Ex-6

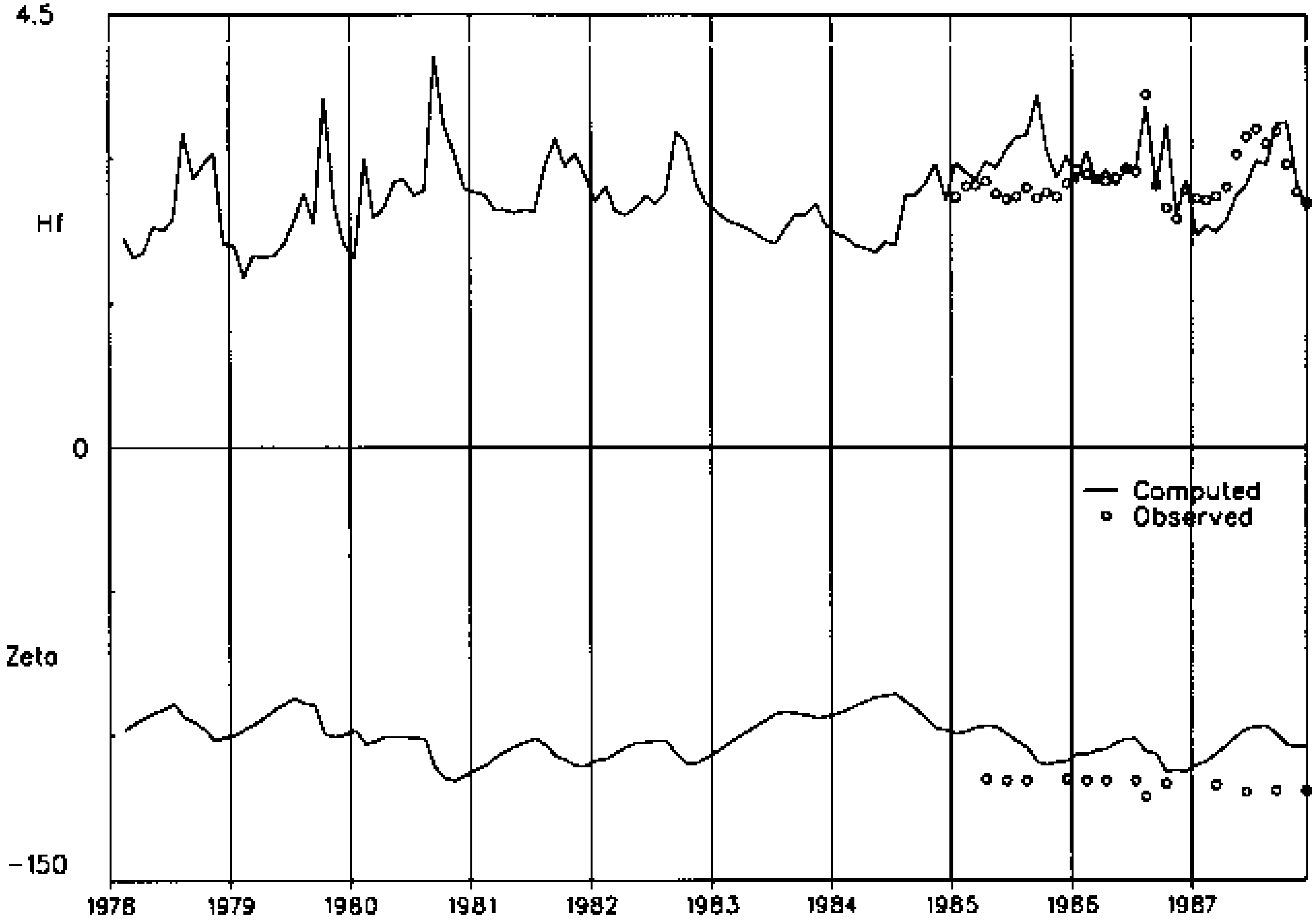

Figure 19. Well water level and Interface depth at Well Ex-10

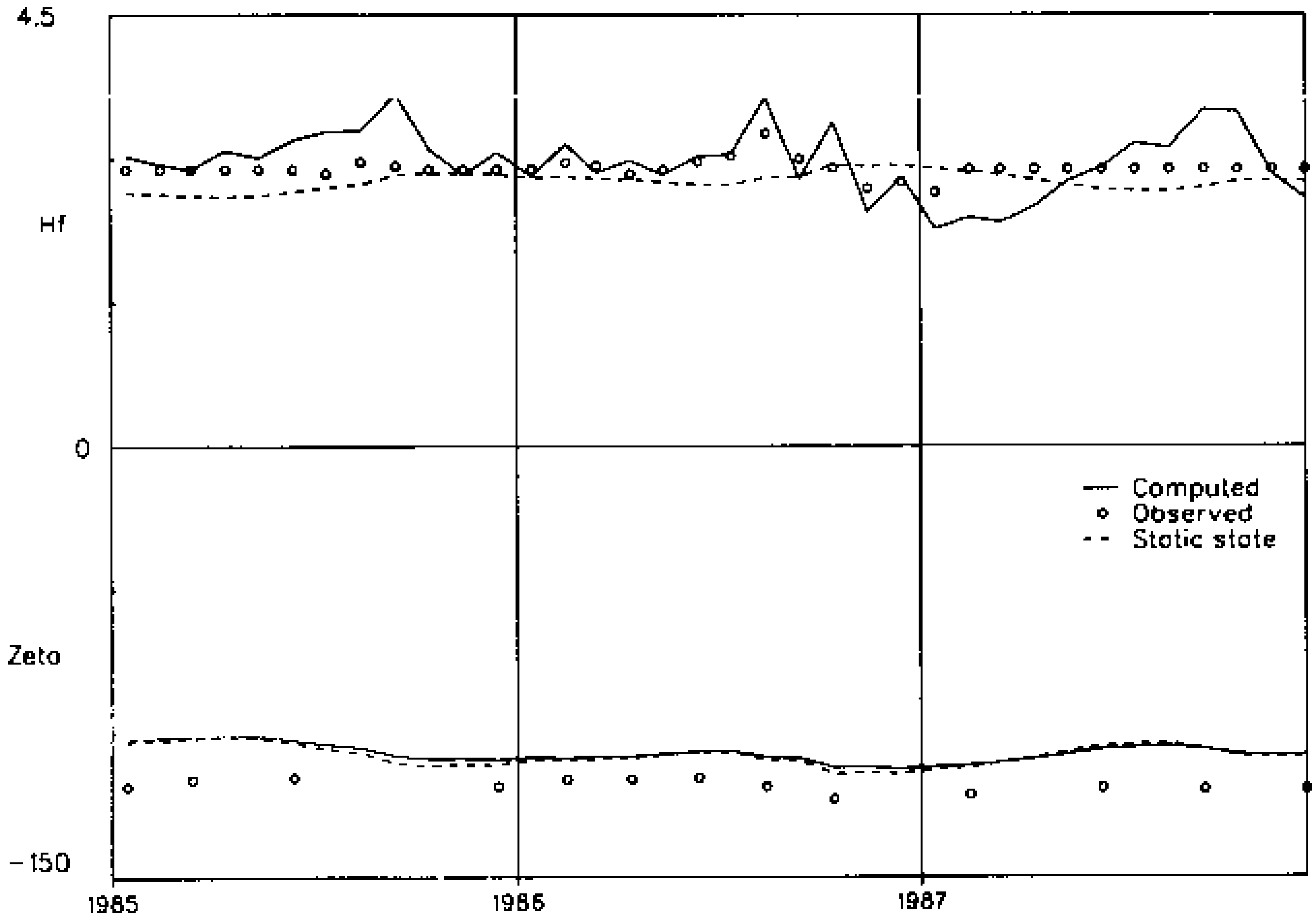

Figure 20. Well water level and Interface depth at Well Ex-9

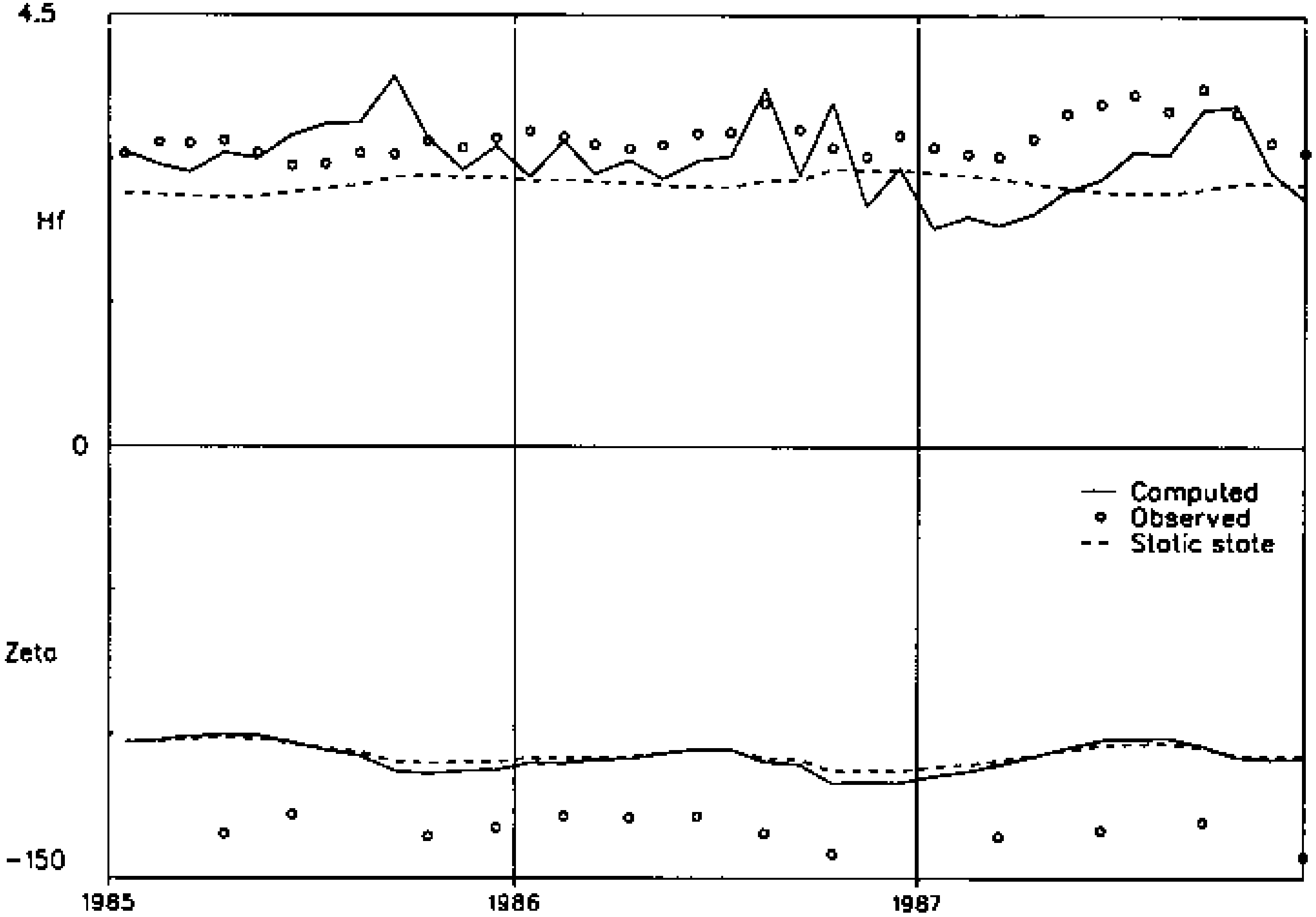

Figure 21. Well water level and Interface depth at Well Ex-7

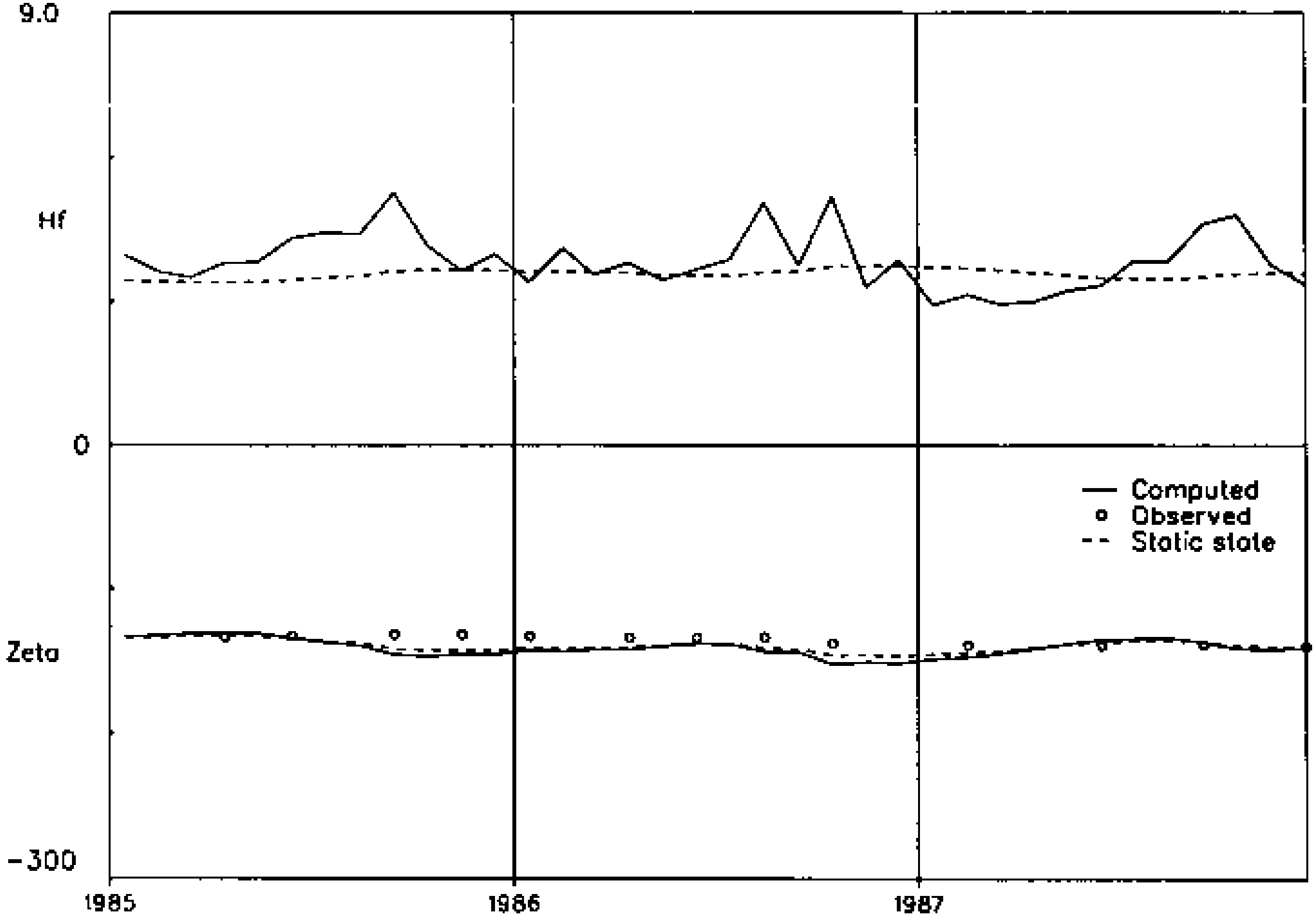

Figure 22. Well water level and Interface depth at Well Ex-6

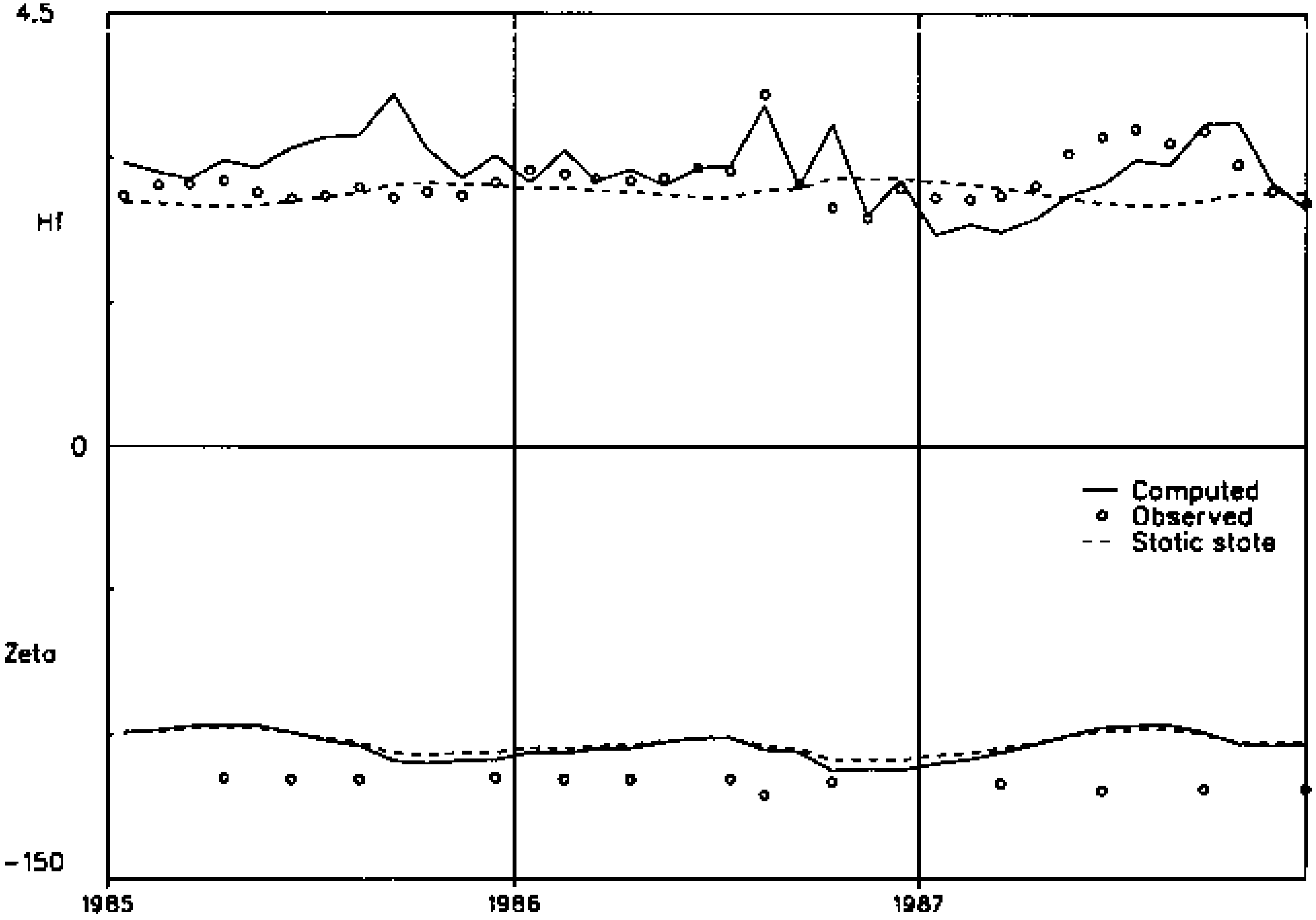

Figure 23. Well water level and interface depth at Well Ex-10

Appendix A

The Pre-processor Program

```
C   THIS PROGRAM RENUMBERS THE NODES TO GET MINIMUM BAND WIDTH.
C   THE SCHEME IS ADOPTED FROM  PAPER BY COLLINS,R.J. IN THE
C   INT. J. NUM. METH. IN ENGG., Vol. 6, pp 345-356 (1973).

        DIMENSION NODE(300,3),NCNT(200),NCN(200,15),NUM(200),NEWNOD(200),
      1           NODOLD(200)

C   NN-TOTAL NO. OF NODES
C   NE--TOTAL NO. OF ELEMENTS
C   NODE(I,J)--NODE NO. OF THE JTH NODE OF ELEMENT I GIVEN ANTICLOCKWISE.

        OPEN(1,FILE='OPTIN.DAT',STATUS='OLD')
        OPEN(2,FILE='OPTOUT.DAT',STATUS='NEW')
        READ(1,*) NN,NE
        READ(1,*) ((NODE(I,J),J=1,3),I=1,NE)
        IBAND=0

C   INITIALIZATION OF VARIABLES
C   NCNT(I)--TOTAL NO. OF NODES CONNECTED TO NODE I
C   NCN(I,J)--NODE NO. OF JTH NODE CONNECTED TO NODE I

        DO 10 I=1,NN
        NCNT(I)=0
        DO 10 J=1,10
10      NCN(I,J)=0
        DO 20 IE=1,NE
        DO 20 I=1,3
        N1=NODE(IE,I)
        DO 20 J=1,3
        IF(I.EQ.J) GO TO 20
        N2=NODE(IE,J)
        IF(NCNT(N1).EQ.0) GO TO 40
        DO 30 K=1,NCNT(N1)
        IF(NCN(N1,K).EQ.N2) GO TO 20
30      CONTINUE
40      IF(ABS(N1-N2).GT.IBAND) IBAND=ABS(N1-N2)
        NCNT(N1)=NCNT(N1)+1
        NCN(N1,NCNT(N1))=N2
20      CONTINUE
        WRITE(2,101) 4*IBAND+3
101     FORMAT(' BAND WIDTH FOR THE ORIGINAL NUMBERING SCHEME IS ',I3//)
        DO 50 I=1,NN

C   INITIALIZATION OF VARIABLES
C   NUM(J)--AN INDEX TO DETERMINE IF NODE J HAS ALREADY
C             BEEN CONSIDERED (NUM=1)  OR NOT (NUM=0)
C   NEWNOD(J)--OLD NODE NO. OF THE NEW NODE J
C   NODOLD(I)--NEW NODE NO. OF THE OLD NODE I

        DO 60 J=1,NN
        NUM(J)=0
60      NEWNOD(J)=0
        INEW=0
        MAX=0
        NEWNOD(1)=I
        NUM(I)=1
        DO 70 J=1,NCNT(I)
        INEW=J+1
        NUM(NCN(I,J))=1
70      NEWNOD(INEW)=NCN(I,J)
        DO 80 J=2,NN
        DO 90 K=1,NCNT(NEWNOD(J))
        IF(NUM(NCN(NEWNOD(J),K)).EQ.1) GO TO 90
        INEW=INEW+1
        NUM(NCN(NEWNOD(J),K))=1
```

```
      NEWNOD(INEW)=NCN(NEWNOD(J),K)
      NDIF=ABS(INEW-J)
      IF(NDIF.GE.IBAND) GO TO 50
      IF(NDIF.GT.MAX) MAX=NDIF
90    CONTINUE
80    CONTINUE
      IBAND=MAX
      DO 100 II=1,NN
100   NODOLD(NEWNOD(II))=II
50    CONTINUE

C  BAND WIDTH FOR 2 DEGREES OF FREEDOM AT EACH NODE

      MBAND=4*IBAND+3
      WRITE(2,102) MBAND
102   FORMAT(' THE OPTIMIZED BAND WIDTH IS ',I3//)
      IF(NODOLD(1).NE.0) THEN
      WRITE(2,103)
103   FORMAT(' THE NEW NODE NOS. OF THE ORIGINAL NODES ARE GIVEN '
     1'BELOW-'/' (OLD NO. FOLLOWED BY THE NEW NO.)'/)
      WRITE(2,104) (I,NODOLD(I),I=1,NN)
104   FORMAT(6(1X,I5,1X,I5,1X)/)
      ELSE
      WRITE(2,105)
105   FORMAT(' THE ORIGINAL SCHEME ITSELF IS OPTIMUM')
      ENDIF
      STOP
      END
```

```
BAND WIDTH FOR THE ORIGINAL NUMBERING SCHEME IS  83


THE CPTIMIZED BAND WIDTH IS  63


THE NEW NODE NOS. OF THE ORIGINAL NODES ARE GIVEN BELOW-
(OLD NO. FOLLOWED BY THE NEW NO.)

      1    24      2    13      3     6      4     2      5     1      6     5

      7    12      8    25      9    14     10     7     11     3     12     4

     13    10     14    20     15    26     16    15     17     8     18     9

     19    11     20    21     21    32     22    27     23    16     24    17

     25    18     26    19     27    22     28    23     29    33     30    34

     31    42     32    28     33    29     34    30     35    31     36    36

     37    43     38    47     39    37     40    38     41    57     42    56

     43    48     44    39     45    40     46    41     47    35     48    44

     49    58     50    49     51    50     52    45     53    46     54    52

     55    51     56    53     57    59     58    60     59    55     60    54

     61    61     62    62     63    68     64    67     65    64     66    66

     67    63     68    72     69    69     70    80     71    70     72    76

     73    65     74    71     75    79     76    73     77    75     78    82

     79    77     80    78     81    85     82    90     83    92     84    86

     85    74     86    81     87    83     88    84     89    91     90    97

     91    87     92    89     93    94     94    98     95    88     96    99

     97    93     98    95     99    96    100   107    101   100    102   101

    103   109    104   102    105   103    106   104    107   105    108   106

    109   117    110   108    111   110    112   111    113   113    114   115

    115   128    116   118    117   119    118   112    119   122    120   114

    121   123    122   116    123   126    124   129    125   120    126   121

    127   132    128   124    129   125    130   127    131   135    132   130

    133   131    134   140    135   133    136   141    137   134    138   136

    139   137    140   138    141   139    142   147    143   142    144   143

    145   144    146   146    147   145    148   148    149   149    150   150

    151   151    152   155    153   153    154   164    155   174    156   156

    157   157    158   167    159   158    160   152    161   159    162   154

    163   161    164   163    165   175    166   179    167   160    168   165
```

169	166	170	162	171	170	172	180	173	187	174	169
175	168	176	171	177	173	178	177	179	178	180	176
181	172	182	181	183	182	184	184	185	185	186	186
187	183	188	189	189	188						

Appendix B

Listing of the Main Program

AVAILABLE AT WERI UPON REQUEST

Appendix C

The User Manual

Introduction

This user manual is intended to help persons solve saltwater intrusion problems using the program SWIGPC. This program and its mathematical basis have been described in Ref. 1. A large scale application of the program is presented in Ref. 2. The model simulates an aquifer in two dimensions (plan), in which a sharp interface separates the fresh water and salt water. A finite element grid of linear triangular elements is used to discretize the aquifer. Appropriate boundary conditions can be specified at the nodes and along the sides of the elements. Any number of pumps can be specified in the network. For these conditions, the model solves for the fresh water and saltwater heads and calculates the depth of the interface, the location of the freshwater and saltwater toes and the velocity in each element.

Program Capabilities

The computer program has the capacity to handle steady and unsteady flows and analyze both confined and unconfined flows. If the aquifer is confined, the program can consider both leaky and non-leaky conditions. Recharge is constant in an element but can be varied from element to element. Specified head or flow conditions can be applied at the boundaries. At a coastal boundary, a mixed or third type boundary condition can be specified. For steady flow conditions, the saltwater head can be specified to be zero along the boundary or at every node in the network. This procedure assures that the Ghyben-Herzberg condition is satisfied. The program can also be run in the unsteady mode with the Ghyben-Herzberg condition.

If the saltwater head at every node in the network is specified to be much less than the anticipated freshwater head, the results of the computer program will show that the thickness of the saltwater layer is equal to an arbitrarily small value (BTOE). Under these conditions, the program can simulate flow in a freshwater aquifer. Use of the program to simulate freshwater aquifers will be limited only by the storage capacity of the computer.

Since the heads are assumed to vary linearly across the triangular element, the velocity in each element will be constant. By specifying NVEL = 1, the program will print out the velocities in the x and y directions in each element in both the freshwater and the saltwater layers. These velocities can then be used to calculate the flow rates across any line or boundary. By specifying NTOE = 1, the program will determine where in the network a freshwater or saltwater toe occurs. A saltwater toe occurs where the interface intersects the lower impermeable boundary. The output provides the element number, the node number, and the fractional distance between the two nodes where the saltwater toe occurs. The same kind of information is also provided about the freshwater toe. A freshwater toe occurs where the phreatic surface intersects the lower impervious boundary or where the interface intersects the upper impervious boundary in confined aquifers.

The program assumes that there are two independent variables at each node: the freshwater head and the saltwater head. After solving for the heads, the depth of the interface ( ) is determined. The location of the interface determines the thickness of the freshwater and the saltwater layers. If, however, the interface is calculated to be below the lower impervious boundary, the entire aquifer thickness will contain only freshwater and the thickness of the saltwater layer should theoretically be zero. However, the program makes the saltwater layer equal to an arbitrarily small value, BTOE. Similarly, when the phreatic surface intersects the lower impervious boundary, the freshwater layer beyond the toe is made equal to BTOE instead of zero. In the course of the program, as the interface and the phreatic surface move with respect to time, the thickness and permeability of the saltwater and the freshwater regions are altered accordingly.

The program uses a weighting factor, O, which varies between zero and one. This factor is useful in regulating the stability and accuracy of the solution. When O = 0, the problem formulation is referred to as explicit. In this formulation, the spatial derivatives are evaluated at the known time step, t. The time step, t, necessary for stable results is very small. This results in very long execution time for the program. When O = 0.5, the problem formulation is referred to as the Crank-Nicolson approximation. This approach provides high accuracy with large values of t, even though the results may show some numerical instability. When O = 1.0, the formulation is known as fully implicit. This formulation provides the maximum stability at a sacrifice of some accuracy. Values of O between 0.5 and 1.0 have been used to provide the proper balance between accuracy and stability. When steady state results are desired, the program should be run with O = 1.0 and t equal to a very large number (e.g. 1.0E20).

The program can be run in any set of consistent units. Thus, if feet and seconds are the length and time units, the permeability and recharge must be input in ft/s and the pump rate in cfs. If meters and days are the length and time units, then the permeability and recharge must be input in m/day and cu.m/day.

When subdividing an aquifer into triangular elements, the versatility of using triangles of different sizes and orientation should be taken advantage of. A node should be placed wherever a pump exists or is projected to be in the future. It is, however, advisable not to let the ratio of the largest triangle to the smallest triangle become too large. It will generally be the size of the smallest triangle that determines the time step t that can be used for stable results.

List of Input Variables

INFILE	Name of the input file, to be typed on screen when prompted [If datafile is on a separate disk, include the path name also, e.g. A:data.dat]
OUTFILE	Name of the file in which output will be stored. To be typed when prompted by the program.
TITLE	Character variable for the title of run.
NDATA	=0, Input data will not be printed out =1, input data will be printed out
NOUT	=1, Output will be printed out at the end of every time counter J. =N, Output will be printed at the end of every Nth time counter J.
JTRC	=0, For not printing detailed matrix output =J, For detailed matrix output to be printed out beginning at time counter J = JTRC
NVEL	=0, For not printing velocities in elements. =1, For print out of magnitude and direction of velocities in elements.
NTOE	=0, For not printing the location of toes. =1, For print out of the location of freshwater and saltwater toes.
NELEV	=0, Nodes are not grouped into elevation groups. =1, Nodes are categorised into NZTYPE groups, each group having the same top and bottom elevation
NPERM	=0, Elements are not grouped into permeability groups. =1, Elements are divided into NKTYPE groups, each having same value of permeability and porosity
NN	Number of nodes.
NE	Number of elements.
NNODHF	Number of nodes with specified freshwater head.
NNODHS	Number of nodes with specified saltwater head.
NPUMPS	Number of pumps in the system.
NEBDQF	Number of element boundaries with specified flow of freshwater.
NEBDQS	Number of element boundaries along which the flow of saltwater is specified.
NEBDCF	Number of element boundaries with coastal boundary condition specified.
NAQTRD	Number of elements which lie below an aquitard.
NRECHG	Number of elements being recharged by specified amount of infiltrated water.
TSTART	Time at the start of the program
DT	Time interval for incrementing time counter, J.
JMAX	Maximum value of time counter J for the execution of the program.
ITERMX	Maximum number of iterations for convergence at each time interval.
TOL	Tolerance (as a fraction) used in convergence of the solution.
THETA	Time weighting factor =0.5 for Crank-Nicolson =1.0 for fully implicit

GAMAF	Specific weight of lighter fluid (freshwater).
GAMAS	Specific weight of denser fluid (saltwater).
BTOE	Thickness of freshwater and saltwater toes.
NODE(I,1) NODE(I,2) NODE(I,3)	Node numbers of element I, specified in counter-clockwise direction.
PFFX(I)	Freshwater permeability in the X-direction in element I.
PFFY(I)	Freshwater permeability in the Y-direction in element I.
PSSX(I)	Saltwater permeability in the X-direction in element I.
PSSY(I)	Saltwater permeability in the Y-direction in element I.
PRSTY(I)	Porosity in element I.
NKTYPE	Total number of groups of elements having same permeabilities and porosity.
PFFXT(I) PFFYT(I) PSSXT(I) PSSYT(I) PRSTYT(I)	Properties for the Ith group of elements.
XC(I)	X-coordinate of node I.
YC(I)	Y-coordinate of node I.
ZB(I)	Elevation of the lower boundary of aquifer.
ZU(I)	Elevation of the upper boundary of aquifer. =1000.1 for unconfined aquifers.
NZTYPE	Number of groups of nodes having same upper and lower aquifer boundary elevations.
ZBT(I) ZUT(I)	Properties for the Ith group of nodes.
IERCHG(I)	Element number of Ith element getting specified recharge.
RECHG(I,J)	Recharge into Ith element (i.e. element number is IERCHG(I)) at time counter J.
IEQF(I)	Element number containing Ith element boundary along which freshwater flow is specified.
NQF(I,1) NQF(I,2)	Nodes between which freshwater flow is specified. Should be given in counter-clockwise direction.
QFWBND(I,J)	Freshwater flow per linear foot between NQF(I,1) and NQF(I,2) entering aquifer between time (J-1) and J.
IEQS(I) NQS(I,1) NQS(I,2) QSWBND(I,J)	Similar to above, for the case of saltwater.
IECF(I) NCF(I,1) NCF(I,2)	Similar to above, for coastal boundary condition.
PC(I)	Permeability of the coastal aquifer adjacent to the Ith boundary.
INODHF(I)	Node number at which freshwater head is specified
SPECHF(I,J)	Specified freshwater head as a function of time.
INODHS(I)	Node number at which saltwater head is specified.
SPECHS(I,J)	Specified saltwater head as a function of time.

NODEP(I)	Node number at which pump is to be located.
NELEM(I)	Number of elements around the pump node.
QPUMP(I,J)	Pump discharge between time counter (J-1) and J.
IEAQTD(I)	Element number of Ith element lying under aquitard
PO(I) PHIO(I) BO(I)	Permeability, head and thickness for aquitard above the Ith element (element no. IEAQTD(I)).
HF(I,1) HS(I,1) BF(I,1) BS(I,1)	Initial conditions for freshwater head, saltwater head, thickness of freshwater layer and thickness of saltwater layer at node number I.
LMNTYP(I)	Type of element I. =0, when all three nodes have freshwater and saltwater thickness greater than BTOE. =1, when all three nodes have freshwater thickness equal to BTOE, but saltwater thickness greater than BTOE. =2, when all three nodes have saltwater thickness equal to BTOE,but freshwater thickness greater than BTOE. =3, when all three nodes have freshwater and saltwater thickness equal to BTOE.
IO(I)	=0, when output is not to be printed at node I. =1, when output is to be printed at node I.

INPUT INSTRUCTIONS

(Free format is used for input except for the title card)

Card 1 TITLE- Maximum 80 characters.
Card 2 NDATA,NOUT,JTRC,NVEL,NTOE,NELEV,NPERM
Card 3 NN, NE, NEBDQF, NEBDQS, NEBDCF, NNODHF, NNODHS,NPUMPS, NAQTRD,NRECHG
Card 4 TSTART,DT,JMAX,ITERMX,TOL,THETA
Card 5 GAMAF,GAMAS,BTOE

ELEMENT DATA

If NPERM = 0 Then

Card Set 6 I,NODE(I,1),NODE(I,2),NODE(I,3),PFFX(I), PFFY(I),PSSX(I),PSSY(I),PRSTY(I)
Repeat in increasing order of element no. I. However, if elements from no. I1 to I2 have same permeabilities and porosity and the node numbers increase linearly, then only I1 and I2 are to be given as input.

If NPERM = 1 Then

Card Set 6
(i) NKTYPE
(ii) PFFXT(I),PFFYT(I),PSSXT(I),PSSYT(I), PRSTYT(I)
Repeat for I = 1 to I = NKTYPE
(iii) I,NODE(I,1),NODE(I,2),NODE(I,3),KTYPE
Repeat in increasing order with similar provision for skipping elements as for the case of NPERM = 0.

NODAL DATA

If NELEV = 0 Then

Card Set 7
(i) I,XC(I),YC(I),ZB(I),ZU(I),INC
[INC indicates that next INC nodes have linearly varying values of XC and YC and same values of ZB and ZU. A 0 has to be input if INC = 0]
If INC = 0
Repeat (i) in increasing order of node numbers for I = 1 to I = NN.
If INC > 0
(ii) I,XC(I),YC(I),ZB(I),ZU(I) For the last node in the series I to (I+INC).

If NELEV = 1 Then

Card Set 7
(i) NZTYPE
(ii) ZBT(I),ZUT(I)
Repeat for I = 1 to I = NZTYPE
(iii) I,XC(I),YC(I),ZTYPE(I),INC
Repeat as before if INC = 0
If INC > 0
(iv) I,XC(I),YC(I) For the last node of the series from I to (I+INC).

```
If NRECHG > 0 Then
     Card Set 8        (i)    IERCHG(I),INC
                       (ii)   RECHG(I,J),J=1,JMAX
                          If INC = 0 Repeat (i) and (ii) for NRECHG
                          elements.
                          If INC > 0
                       (iii) IERCHG(I+1) to IERCHG(I+INC)  (i.e.the
                             element numbers of  the  INC  elements
                             having  the   same   recharge  as  the
                             current element)
                          If INC < 0
                       (iii) IECR    ( all elements from the current
                                       no. to element no. IECR  will
                                       have the same recharge)
If NEBDQF > 0 Then
     Card Set 9        (i)    IEQF(I),NQF(I,1),NQF(I,2)
                       (ii)   QFWBND(I,J),J=1,JMAX
                              Repeat (i) and (ii) till I=NEBDQF
If NEBDQS > 0 Then
     Card Set 10       (i)    IEQS(I),NQS(I,1),NQS(I,2)
                       (ii)   QSWBND(I,J),J=1,JMAX
                              Repeat (i) and (ii) till I=NEBDQS
If NEBDCF > 0 Then
     Card Set 11       (i)    IECF(I),NCF(I,1),NCF(I,2),PC(I)
                              Repeat till I=NEBDCF
If NNODHF > 0 Then
     Card Set 12       (i)    INODHF(I)
                       (ii)   SPECHF(I,J),J=1,JMAX
                              Repeat (i) and (ii) till I=NNODHF
If NNODHS > 0 Then
     Card Set 13       (i)    INODHS(I)
                       (ii)   SPECHS(I,J),J=1,JMAX
                              Repeat (i) and (ii) till I=NNODHS
If NPUMPS > 0 Then
     Card Set 14       (i)    NODEP(I),NELEM(I)
                       (ii)   QPUMP(I,J),J=1,JMAX
                              Repeat (i) and (ii) till I=NPUMPS
                       INITIAL CONDITIONS
                       ------------------
Card Set 15     I,HF(I,1),HS(I,1),BF(I,1),BS(I,1)
                Repeat in increasing order of node number,  but if
                a series of consecutive nodes have same values  of
                HF,HS,BF  and BS input only the first node of  the
                series.[ However, the last node of the grid ,i.e.,
                node number NN has to be given as input even  when
                it is in a series of similar values ]
If NAQTRD > 0 Then
     Card Set 16       (i)  IEAQTD(I),PO(I),PHIO(I),BO(I),INC
                              Repeat as per instructions for Card Set 8.
Card Set 17     LMNTYP(I),I=1,NE
                Values separated by blanks or comma.
Card Set 18     IO(I),I=1,NN
                Values separated by blanks or comma.
```

Appendix D

Finite Element Matrices

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Appendix E

Sample Data Input and Output for 1-D Problem

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Appendix F

Input Data for Guam Aquifer

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